Mathematical and Computer Modelling 53 (2011) 1664–1669

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Least squares based iterative parameter estimation algorithm for multivariable controlled ARMA system modelling with finite measurement data[☆]

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ARTICLE INFO

Article history:
Received 23 October 2010
Received in revised form 15 December 2010
Accepted 15 December 2010

Keywords:
System modelling
Recursive identification
Iterative identification
Parameter estimation
Least squares
Multivariable systems

ABSTRACT

Difficulties of identification for multivariable controlled autoregressive moving average (ARMA) systems lie in that there exist unknown noise terms in the information vector, and the iterative identification can be used for the system with unknown terms in the information vector. By means of the hierarchical identification principle, those noise terms in the information vector are replaced with the estimated residuals and a least squares based iterative algorithm is proposed for multivariable controlled ARMA systems. The simulation results indicate that the proposed algorithm is effective.

1. Introduction

System identification is an important approach to model dynamical systems and has been used in many areas such as chemical processes [1], and signal processing [2]. Several methods have been developed for system identification, e.g., the least squares methods [3], gradient based methods [4], the maximum likelihood methods [5] and the step response based method [6,7]. Some useful techniques are used in system identification. For example, the polynomial transformation technique is used to deal with the dual-rate sampled-data systems and the systems with missing observations [8]; the auxiliary model identification idea is used to handle the cases that the information vector contains unknown intermediate variables [9–11]; the hierarchical identification principle is used to reduce the computational cost [12–15]; the multi-innovation identification theory [16–27] and the iterative identification method [11,28–31] make sufficient use of all input–output data and can improve the parameter estimation accuracy.

The least squares based and gradient based iterative methods have been used to solve some matrix equations [32–42]. Also, the iterative methods are very useful for system identification, e.g., Ding et al. proposed a least squares based and a gradient based iterative identification method for OE and OEMA systems [11], and presented a least squares based iterative algorithm for Hammerstein nonlinear ARMAX systems [28]. Liu et al. developed a least squares based iterative identification method for a class of multirate sampled-data systems [31]. Han et al. gave a hierarchical least squares based iterative identification algorithm for a class of multivariable CARMA-like systems [15]. In this paper, we propose a least squares

☆ This work was supported in part by the National Natural Science Foundation of China.
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0895-7177/$ – see front matter © 2011 Published by Elsevier Ltd
doi:10.1016/j.mcm.2010.12.034

based iterative identification method for multivariable controlled ARMA systems. The multivariable model considered in this paper is different from the model in [15].

The rest of this paper is organized as follows. Section 2 derives a least squares based iterative algorithm for the multivariable controlled ARMA systems and gives the identification steps in detail. Section 3 provides a simulation example to show the effectiveness of the proposed algorithm. Finally, concluding remarks are given in Section 4.

2. The derivation of identification algorithm

Consider a multivariable system described by the following controlled ARMA model (multivariable CARMA model for short),

$$\boldsymbol{A}(z)\boldsymbol{y}(t) = \boldsymbol{B}(z)\boldsymbol{u}(t) + \boldsymbol{D}(z)\boldsymbol{v}(t), \tag{1}$$

where $\boldsymbol{u}(t) = [u_1(t), u_2(t), \ldots, u_r(t)]^{\mathrm{T}} \in \mathbb{R}^r$ is the system input vector, $\boldsymbol{y}(t) = [y_1(t), y_2(t), \ldots, y_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ the system output vector and $\boldsymbol{v}(t) = [v_1(t), v_2(t), \ldots, v_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ the white noise vector with zero mean, z^{-1} is a unit delay operator: $z^{-1}\boldsymbol{y}(t) = \boldsymbol{y}(t-1)$, $\boldsymbol{A}(z)$, $\boldsymbol{B}(z)$ and $\boldsymbol{D}(z)$ are matrix-coefficient polynomials in z^{-1} with degrees n_a, n_b and n_d, respectively, and

$$\begin{aligned}
\boldsymbol{A}(z) &= \boldsymbol{I} + \boldsymbol{A}_1 z^{-1} + \boldsymbol{A}_2 z^{-2} + \cdots + \boldsymbol{A}_{n_a} z^{-n_a},\\
\boldsymbol{B}(z) &= \boldsymbol{B}_1 z^{-1} + \boldsymbol{B}_2 z^{-2} + \cdots + \boldsymbol{B}_{n_b} z^{-n_b},\\
\boldsymbol{D}(z) &= \boldsymbol{I} + \boldsymbol{D}_1 z^{-1} + \boldsymbol{D}_2 z^{-2} + \cdots + \boldsymbol{D}_{n_d} z^{-n_d}.
\end{aligned}$$

$\boldsymbol{A}_i \in \mathbb{R}^{m\times m}$, $\boldsymbol{B}_i \in \mathbb{R}^{m\times r}$ and $\boldsymbol{D}_i \in \mathbb{R}^{m\times m}$ are the matrix coefficients to be estimated. Assume that the orders n_a, n_b and n_d are known and $\boldsymbol{u}(t) = \boldsymbol{0}$, $\boldsymbol{y}(t) = \boldsymbol{0}$ and $\boldsymbol{v}(t) = \boldsymbol{0}$ as $t \leqslant 0$.

The goal of this paper is to present an iterative algorithm to estimate the matrices $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ and $\boldsymbol{D}_i$ from the measured inputs and outputs $\{u(t), y(t) : t = 1, 2, \ldots, L\}$ (L denotes the data length), using the least squares principle.

Let T be the matrix transpose. Define the parameter matrix $\boldsymbol{\theta}$ and the information vector $\boldsymbol{\varphi}(t)$ as

$$\begin{aligned}
\boldsymbol{\theta}^{\mathrm{T}} &:= [\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_{n_a}, \boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_{n_b}, \boldsymbol{D}_1, \boldsymbol{D}_2, \ldots, \boldsymbol{D}_{n_d}] \in \mathbb{R}^{m\times n},\\
\boldsymbol{\varphi}(t) &:= [-\boldsymbol{y}^{\mathrm{T}}(t-1), -\boldsymbol{y}^{\mathrm{T}}(t-2), \ldots, -\boldsymbol{y}^{\mathrm{T}}(t-n_a), \boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \ldots, \boldsymbol{u}^{\mathrm{T}}(t-n_b),\\
&\qquad \boldsymbol{v}^{\mathrm{T}}(t-1), \boldsymbol{v}^{\mathrm{T}}(t-2), \ldots, \boldsymbol{v}^{\mathrm{T}}(t-n_d)]^{\mathrm{T}} \in \mathbb{R}^n,\ \ n := mn_a + rn_b + mn_d
\end{aligned} \tag{2}$$

then the system model in (1) can be equivalently written as

$$\boldsymbol{y}(t) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(t) + \boldsymbol{v}(t). \tag{3}$$

Eq. (3) is the identification model for the multivariable system in (1).

Consider the data from $t = 1$ to $t = L$, and define the stacked output matrix $\boldsymbol{Y}(L)$, the stacked information matrix $\boldsymbol{\Phi}(L)$ and the stacked white noise matrix $\boldsymbol{V}(L)$ as

$$\begin{aligned}
\boldsymbol{Y}(L) &:= [\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots, \boldsymbol{y}(L)] \in \mathbb{R}^{m\times L},\\
\boldsymbol{\Phi}(L) &:= [\boldsymbol{\varphi}(1), \boldsymbol{\varphi}(2), \ldots, \boldsymbol{\varphi}(L)] \in \mathbb{R}^{n\times L},\\
\boldsymbol{V}(L) &:= [\boldsymbol{v}(1), \boldsymbol{v}(2), \ldots, \boldsymbol{v}(L)] \in \mathbb{R}^{m\times L}.
\end{aligned}$$

Note that $\boldsymbol{Y}(L)$ and $\boldsymbol{\Phi}(L)$ contain all the measured data $\{u(t), y(t) : t = 1, 2, \ldots, L\}$. From (3), we have

$$\boldsymbol{Y}(L) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\Phi}(L) + \boldsymbol{V}(L). \tag{4}$$

Define a quadratic criterion function:

$$\boldsymbol{J}(\boldsymbol{\theta}) := \|\boldsymbol{Y}(L) - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\Phi}(L)\|^2, \qquad \|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]. \tag{5}$$

Note that $\boldsymbol{V}(L)$ is a white noise matrix with zero mean. For the optimization problem in (5), minimizing $\boldsymbol{J}(\boldsymbol{\theta})$ and letting its partial derivative with respect to $\boldsymbol{\theta}$ be zero give

$$\frac{\partial \boldsymbol{J}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = -2[\boldsymbol{Y}(L) - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\Phi}(L)]\boldsymbol{\Phi}^{\mathrm{T}}(L) = \boldsymbol{0}.$$

Assume that the information vector $\boldsymbol{\varphi}(t)$ is persistently exciting, that is, $[\boldsymbol{\Phi}(L)\boldsymbol{\Phi}^{\mathrm{T}}(L)]$ is an invertible matrix, then from above equation, we can obtain the least squares estimate (LSE) of $\boldsymbol{\theta}$:

$$\begin{aligned}
\hat{\boldsymbol{\theta}} &= [\boldsymbol{\Phi}(L)\boldsymbol{\Phi}^{\mathrm{T}}(L)]^{-1}\boldsymbol{\Phi}(L)\boldsymbol{Y}^{\mathrm{T}}(L)\\
&= \left[\sum_{t=1}^{L}\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t)\right]^{-1}\sum_{t=1}^{L}\boldsymbol{\varphi}(t)\boldsymbol{y}^{\mathrm{T}}(t).
\end{aligned} \tag{6}$$

However, from (2), we can see that the information vector $\boldsymbol{\varphi}(t)$ $(t = 1, 2, \ldots, L)$ contain the unmeasurable noise terms $\boldsymbol{v}(t-i)$ $(i = 1, 2, \ldots, n_d)$, thus Eq. (6) cannot give the estimate $\hat{\boldsymbol{\theta}}$ directly. The commonly used method is to replace the unmeasurable noise terms with their estimated residuals. In this paper, we propose an iterative identification method using the hierarchical identification principle. Let $k = 1, 2, 3, \ldots$ be an iteration variable, and $\hat{\boldsymbol{\theta}}_k$ be the estimate of $\boldsymbol{\theta}$ at iteration k, $\hat{\boldsymbol{\varphi}}_k(t)$ denote the information vector obtained by replacing the inner unknown $\boldsymbol{v}(t-i)$ in $\boldsymbol{\varphi}(t)$ with the estimate $\hat{\boldsymbol{v}}_{k-1}(t-i)$ at iteration $k-1$, and $\hat{\boldsymbol{\Phi}}_k(L)$ denote the stacked information matrix obtained by replacing $\boldsymbol{\varphi}(t)$ in $\boldsymbol{\Phi}(L)$ with $\hat{\boldsymbol{\varphi}}_k(t)$, i.e.,

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_k(t) &:= [-\boldsymbol{y}^{\mathrm{T}}(t-1), -\boldsymbol{y}^{\mathrm{T}}(t-2), \ldots, -\boldsymbol{y}^{\mathrm{T}}(t-n_a), \boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \ldots, \boldsymbol{u}^{\mathrm{T}}(t-n_b),\\ &\quad\ \hat{\boldsymbol{v}}_{k-1}^{\mathrm{T}}(t-1), \hat{\boldsymbol{v}}_{k-1}^{\mathrm{T}}(t-2), \ldots, \hat{\boldsymbol{v}}_{k-1}^{\mathrm{T}}(t-n_d)]^{\mathrm{T}} \in \mathbb{R}^n,\\ \hat{\boldsymbol{\Phi}}_k(L) &:= [\hat{\boldsymbol{\varphi}}_k(1), \hat{\boldsymbol{\varphi}}_k(2), \ldots, \hat{\boldsymbol{\varphi}}_k(L)] \in \mathbb{R}^{n\times L}.\end{aligned} \tag{7}$$

From (3), we have

$$\boldsymbol{v}(t) = \boldsymbol{y}(t) - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\varphi}(t).$$

If $\boldsymbol{\varphi}(t)$ and $\boldsymbol{\theta}$ are replaced with their estimates $\hat{\boldsymbol{\varphi}}_k(t)$ and $\hat{\boldsymbol{\theta}}_k$, then the estimate of $\boldsymbol{v}(t)$ at iteration k can be computed by

$$\hat{\boldsymbol{v}}_k(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{\theta}}_k^{\mathrm{T}}\hat{\boldsymbol{\varphi}}_k(t). \tag{8}$$

Replacing $\boldsymbol{\Phi}(L)$ in (6) with $\hat{\boldsymbol{\Phi}}_k(L)$ gives the least squares based iterative parameter estimation algorithm for the multivariable CARMA systems (CARMA-LSI):

$$\hat{\boldsymbol{\theta}}_k = [\hat{\boldsymbol{\Phi}}_k(L)\hat{\boldsymbol{\Phi}}_k^{\mathrm{T}}(L)]^{-1}\hat{\boldsymbol{\Phi}}_k(L)\boldsymbol{Y}^{\mathrm{T}}(L), \quad k = 1, 2, 3, \ldots \tag{9}$$

$$\hat{\boldsymbol{\Phi}}_k(L) = [\hat{\boldsymbol{\varphi}}_k(1), \hat{\boldsymbol{\varphi}}_k(2), \ldots, \hat{\boldsymbol{\varphi}}_k(L)], \tag{10}$$

$$\boldsymbol{Y}(L) = [\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots, \boldsymbol{y}(L)], \tag{11}$$

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_k(t) &= [-\boldsymbol{y}^{\mathrm{T}}(t-1), -\boldsymbol{y}^{\mathrm{T}}(t-2), \ldots, -\boldsymbol{y}^{\mathrm{T}}(t-n_a), \boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2), \ldots, \boldsymbol{u}^{\mathrm{T}}(t-n_b),\\ &\quad\ \hat{\boldsymbol{v}}_{k-1}^{\mathrm{T}}(t-1), \hat{\boldsymbol{v}}_{k-1}^{\mathrm{T}}(t-2), \ldots, \hat{\boldsymbol{v}}_{k-1}^{\mathrm{T}}(t-n_d)]^{\mathrm{T}},\end{aligned} \tag{12}$$

$$\hat{\boldsymbol{v}}_k(t) = \boldsymbol{y}(t) - \hat{\boldsymbol{\theta}}_k^{\mathrm{T}}\hat{\boldsymbol{\varphi}}_k(t), \quad t = 1, 2, \ldots, L. \tag{13}$$

In this algorithm, the initial value $\hat{\boldsymbol{v}}_0(t)$ is often chosen as a random vector. From (9)–(13), we can see that the CARMA-LSI algorithm performs a hierarchical interactive process: when computing the parameter estimates $\hat{\boldsymbol{\theta}}_k$, the unknown noise terms $\boldsymbol{v}(t-i)$, $i = 1, 2, \ldots, n_d$, in the information vector are replaced with their corresponding estimates $\hat{\boldsymbol{v}}_{k-1}(t-i)$ at the $k-1$th iteration, while the noise estimates $\hat{\boldsymbol{v}}_k(t)$ at iteration k are computed from the parameter estimates $\hat{\boldsymbol{\theta}}_k$.

The identification steps of the CARMA-LSI algorithm to compute $\hat{\boldsymbol{\theta}}_k(t)$ are listed as follows

1. Collect the input–output data $\{u(t), y(t) : t = 1, 2, \ldots, L\}(L \gg n)$ and form the stacked output matrix $\boldsymbol{Y}(L)$ by (11).
2. Let $k = 1$, set $\hat{\boldsymbol{v}}_0(t)$ a random vector.
3. Form $\hat{\boldsymbol{\varphi}}_k(t)$ by (12), and then form $\hat{\boldsymbol{\Phi}}_k(L)$ by (10).
4. Update the estimate $\hat{\boldsymbol{\theta}}_k$ by (9).
5. Compute $\hat{\boldsymbol{v}}_k(t)$ by (13).
6. Compare $\hat{\boldsymbol{\theta}}_k$ with $\hat{\boldsymbol{\theta}}_{k-1}$, if they are sufficiently close, or for some pre-set small ε, if

$$\|\hat{\boldsymbol{\theta}}_k - \hat{\boldsymbol{\theta}}_{k-1}\| \leqslant \varepsilon$$

then terminate this procedure and obtain the iterative time k and estimate $\hat{\boldsymbol{\theta}}_k$; otherwise, increase k by 1 and go to step 3.

The flowchart of computing the parameter estimate $\hat{\boldsymbol{\theta}}_k$ is shown in Fig. 1.

3. Example

In this section, an example is given to show that the proposed iterative algorithm is effective. Consider the following 2-input and 2-output system:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0.60 & 0.50 \\ -0.80 & 1.00 \end{bmatrix}\begin{bmatrix} y_1(t-1) \\ y_2(t-1) \end{bmatrix} = \begin{bmatrix} 1.50 & -0.40 \\ -0.50 & 1.10 \end{bmatrix}\begin{bmatrix} u_1(t-1) \\ u_2(t-1) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} 0.20 & -0.10 \\ -0.10 & 0.60 \end{bmatrix}\begin{bmatrix} v_1(t-1) \\ v_2(t-1) \end{bmatrix}.$$

Here, $\{u_1(t)\}$ and $\{u_2(t)\}$ are taken as persistent excitation signal sequences with zero mean and unit variance, $\{v_1(t)\}$ and $\{v_2(t)\}$ as white noise sequences with zero mean and variances $\sigma_1^2 = \sigma_2^2 = 0.50^2$. Applying the CARMA-LSI algorithm

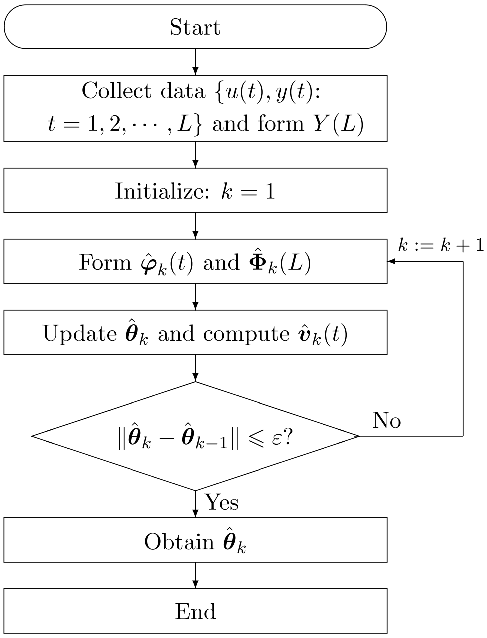

Fig. 1. The flowchart of computing the CARMA-LSI parameter estimate $\hat{\boldsymbol{\theta}}_k$.

Table 1
The CARMA-LSI parameter estimates and errors ($L = 1000$).

k	1	2	3	4	5	6
$a_{11} = 0.60000$	0.59989	0.59991	0.59991	0.59991	0.59991	0.59992
$a_{12} = 0.50000$	0.50009	0.50007	0.50007	0.50007	0.50007	0.50007
$b_{11} = 1.50000$	1.53201	1.52962	1.53039	1.53052	1.53053	1.53053
$b_{12} = -0.40000$	−0.40837	−0.40712	−0.40649	−0.40593	−0.40584	−0.40583
$d_{11} = 0.20000$	0.00440	0.21978	0.23909	0.24087	0.24088	0.24086
$d_{12} = -0.10000$	0.01175	−0.07751	−0.11764	−0.12421	−0.12427	−0.12421
$a_{21} = -0.80000$	−0.79959	−0.79963	−0.79963	−0.79962	−0.79962	−0.79964
$a_{22} = 1.00000$	0.99959	0.99964	0.99963	0.99963	0.99964	0.99963
$b_{21} = -0.50000$	−0.48619	−0.48893	−0.48960	−0.48993	−0.48998	−0.48998
$b_{22} = 1.10000$	1.11032	1.09918	1.09497	1.09308	1.09277	1.09276
$d_{21} = -0.10000$	−0.01132	−0.07535	−0.11506	−0.12093	−0.12093	−0.12092
$d_{22} = 0.60000$	0.01841	0.43624	0.57962	0.60221	0.60238	0.60236
δ (%)	24.67664	6.70177	2.34000	2.41533	2.41777	2.41642

Table 2
The CARMA-LSI parameter estimates and errors ($L = 2000$).

k	1	2	3	4	5	6
$a_{11} = 0.60000$	0.59990	0.59993	0.59993	0.59992	0.59993	0.59993
$a_{12} = 0.50000$	0.50033	0.50031	0.50031	0.50032	0.50031	0.50031
$b_{11} = 1.50000$	1.52074	1.51557	1.51486	1.51460	1.51456	1.51455
$b_{12} = -0.40000$	−0.39568	−0.39719	−0.39705	−0.39683	−0.39679	−0.39678
$d_{11} = 0.20000$	0.01671	0.19748	0.21280	0.21364	0.21360	0.21360
$d_{12} = -0.10000$	−0.01715	−0.07863	−0.11148	−0.11515	−0.11521	−0.11521
$a_{21} = -0.80000$	−0.79991	−0.79994	−0.79995	−0.79993	−0.79993	−0.79994
$a_{22} = 1.00000$	0.99994	1.00000	0.99999	0.99999	1.00000	1.00000
$b_{21} = -0.50000$	−0.49558	−0.48877	−0.48603	−0.48500	−0.48483	−0.48481
$b_{22} = 1.10000$	1.09733	1.09810	1.09686	1.09611	1.09596	1.09594
$d_{21} = -0.10000$	−0.02135	−0.08146	−0.10545	−0.11015	−0.11043	−0.11045
$d_{22} = 0.60000$	0.02569	0.40962	0.52741	0.54755	0.54946	0.54963
δ (%)	24.00801	7.56540	3.03650	2.38901	2.32915	2.32408

in (9)–(13) to estimate the parameter matrix $\boldsymbol{\theta}$ of this system. The parameter estimates and their estimation errors with the data length $L = 1000$, $L = 2000$ and $L = 3000$ are shown in Tables 1–3, where the estimation error is defined as $\delta := \|\hat{\boldsymbol{\theta}}_k - \boldsymbol{\theta}\| / \|\boldsymbol{\theta}\|$.

Table 3
The CARMA-LSI parameter estimates and errors ($L = 3000$).

k	1	2	3	4	5	6
$a_{11} = 0.60000$	0.60005	0.60005	0.60005	0.60005	0.60005	0.60005
$a_{12} = 0.50000$	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
$b_{11} = 1.50000$	1.50744	1.50452	1.50436	1.50425	1.50423	1.50423
$b_{12} = -0.40000$	−0.41562	−0.41347	−0.41354	−0.41342	−0.41340	−0.41340
$d_{11} = 0.20000$	0.01038	0.16975	0.18386	0.18587	0.18611	0.18613
$d_{12} = -0.10000$	0.00825	−0.05506	−0.08387	−0.09068	−0.09126	−0.09127
$a_{21} = -0.80000$	−0.79995	−0.79996	−0.79997	−0.79996	−0.79996	−0.79997
$a_{22} = 1.00000$	0.99992	0.99994	0.99994	0.99994	0.99994	0.99994
$b_{21} = -0.50000$	−0.51447	−0.50962	−0.50968	−0.50934	−0.50928	−0.50928
$b_{22} = 1.10000$	1.09646	1.09933	1.09930	1.09869	1.09858	1.09858
$d_{21} = -0.10000$	−0.01091	−0.05595	−0.08684	−0.09168	−0.09193	−0.09194
$d_{22} = 0.60000$	0.00119	0.42970	0.56287	0.57790	0.57801	0.57802
δ (%)	25.18160	7.22840	1.90336	1.31543	1.29898	1.29799

From the simulation results in Tables 1–3, we can draw the following conclusions:

1. The estimation errors δ are becoming smaller (in general) as the iterations k increases. Thus the proposed algorithm for multivariable CARMA systems is effective.
2. A longer data length L leads to a smaller estimation error under the same noise level.
3. The CARMA-LSI algorithm converges very fast and needs only a few iterations to converge to their true values.

4. Conclusions

This paper presents a least squares based iterative parameter estimation algorithm for multivariable controlled ARMA systems. The basic idea is to use the iterative technique and to replace the unknown terms in the information vector with their iterative estimates. Since the proposed algorithm makes full use of the measured input–output data, it can provide more accurate parameter estimates than existing recursive algorithms. The proposed algorithm can be extended to identify time-varying systems [43], nonlinear systems [44–48], dual-rate/multirate systems [49–58], as well as to design filters [59–62] and estimate states [63].

References

[1] B. Pekta, Modeling and computer simulation of the identification problem related to the sludge concentration in a settler, Mathematical and Computer Modelling 49 (5–6) (2009) 843–855.
[2] Y.N. Cao, Z.Q. Liu, Signal frequency and parameter estimation for power systems using the hierarchical dentification principle, Mathematical and Computer Modelling 51 (5–6) (2010) 854–861.
[3] L. Ljung, System Identification: Theory for the User, Prentice-Hall, Englewood Cliffs, NJ, 1999.
[4] Y.J. Liu, J. Sheng, R.F. Ding, Convergence of stochastic gradient algorithm for multivariable ARX-like systems, Computers & Mathematics with Applications 59 (8) (2010) 2615–2627.
[5] J.C. Agüro, J.I. Yuz, G.C. Goodwin, R.A. Delgado, On the equivalence of time and frequency domain maximum likelihood estimation, Automatica 46 (2) (2010) 260–270.
[6] S. Ahmed, B. Huang, S.L. Shah, Novel identification method from step response, Control Engineering Practice 15 (5) (2007) 545–556.
[7] S. Ahmed, B. Huang, S.L. Shah, Identification from step responses with transient initial conditions, Journal of Process Control 18 (2) (2008) 121–130.
[8] J. Ding, L.L. Han, X.M. Chen, Time series AR modeling with missing observations based on the polynomial transformation, Mathematical and Computer Modelling 51 (5–6) (2010) 527–536.
[9] Y.J. Liu, Y.S. Xiao, X.L. Zhao, Multi-innovation stochastic gradient algorithm for multiple-input single-output systems using the auxiliary model, Applied Mathematics and Computation 215 (4) (2009) 1477–1483.
[10] F. Ding, J. Ding, Least squares parameter estimation with irregularly missing data, International Journal of Adaptive Control and Signal Processing 24 (7) (2010) 540–553.
[11] F. Ding, P.X. Liu, G. Liu, Gradient based and least-squares based iterative identification methods for OE and OEMA systems, Digital Signal Processing 20 (3) (2010) 664–677.
[12] L.L. Xiang, L.B. Xie, Y.W. Liao, R.F. Ding, Hierarchical least squares algorithms for single-input multiple-output systems based on the auxiliary model, Mathematical and Computer Modelling 52 (5–6) (2010) 918–924.
[13] F. Ding, T. Chen, Hierarchical gradient-based identification of multivariable discrete-time systems, Automatica 41 (2) (2005) 315–325.
[14] F. Ding, T. Chen, Hierarchical least squares identification methods for multivariable systems, IEEE Transactions on Automatic Control 50 (3) (2005) 397–402.
[15] H.Q. Han, L. Xie, F. Ding, X.G. Liu, Hierarchical least-squares based iterative identification for multivariable systems with moving average noises, Mathematical and Computer Modelling 51 (9–10) (2010) 1213–1220.
[16] F. Ding, T. Chen, Performance analysis of multi-innovation gradient type identification methods, Automatica 43 (1) (2007) 1–14.
[17] F. Ding, Several multi-innovation identification methods, Digital Signal Processing 20 (4) (2010) 1027–1039.
[18] D.Q. Wang, F. Ding, Performance analysis of the auxiliary models based multi-innovation stochastic gradient estimation algorithm for output error systems, Digital Signal Processing 20 (3) (2010) 750–762.
[19] D.Q. Wang, Y.Y. Chu, F. Ding, Auxiliary model-based RELS and MI-ELS algorithms for Hammerstein OEMA systems, Computers & Mathematics with Applications 59 (9) (2010) 3092–3098.
[20] L.L. Han, F. Ding, Identification for multirate multi-input systems using the multi-innovation identification theory, Computers & Mathematics with Applications 57 (9) (2009) 1438–1449.
[21] L.L. Han, F. Ding, Multi-innovation stochastic gradient algorithms for multi-input multi-output systems, Digital Signal Processing 19 (4) (2009) 545–554.

[22] J.B. Zhang, F. Ding, Y. Shi, Self-tuning control based on multi-innovation stochastic gradient parameter estimation, Systems & Control Letters 58 (1) (2009) 69–75.
[23] F. Ding, P.X. Liu, G. Liu, Auxiliary model based multi-innovation extended stochastic gradient parameter estimation with colored measurement noises, Signal Processing 89 (10) (2009) 1883–1890.
[24] L. Xie, H.Z. Yang, F. Ding, Modeling and identification for non-uniformly periodically sampled-data systems, IET Control Theory & Applications 4 (5) (2010) 784–794.
[25] F. Ding, P.X. Liu, G. Liu, Multi-innovation least squares identification for system modeling, IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 40 (3) (2010) 767–778.
[26] Y.J. Liu, L. Yu, F. Ding, Multi-innovation extended stochastic gradient algorithm and its performance analysis, Circuits, Systems and Signal Processing 29 (4) (2010) 649–667.
[27] J. Chen, Y. Zhang, R.F. Ding, Auxiliary model based multi-innovation algorithms for multivariable nonlinear systems, Mathematical and Computer Modelling 52 (9–10) (2010) 1428–1434.
[28] F. Ding, T. Chen, Identification of Hammerstein nonlinear ARMAX systems, Automatica 41 (9) (2005) 1479–1489.
[29] Y.J. Liu, D.Q. Wang, F. Ding, Least-squares based iterative algorithms for identifying Box–Jenkins models with finite measurement data, Digital Signal Processing 20 (5) (2010) 1458–1467.
[30] D.Q. Wang, G.W. Yang, R.F. Ding, Gradient-based iterative parameter estimation for Box–Jenkins systems, Computers & Mathematics with Applications 60 (5) (2010) 1200–1208.
[31] X.G. Liu, J. Lu, Least squares based iterative identification for a class of multirate systems, Automatica 46 (3) (2010) 549–554.
[32] F. Ding, T. Chen, Gradient based iterative algorithms for solving a class of matrix equations, IEEE Transactions on Automatic Control 50 (8) (2005) 1216–1221.
[33] F. Ding, T. Chen, Iterative least squares solutions of coupled Sylvester matrix equations, Systems & Control Letters 54 (2) (2005) 95–107.
[34] F. Ding, T. Chen, On iterative solutions of general coupled matrix equations, SIAM Journal on Control and Optimization 44 (6) (2006) 2269–2284.
[35] F. Ding, P.X. Liu, J. Ding, Iterative solutions of the generalized Sylvester matrix equations by using the hierarchical identification principle, Applied Mathematics and Computation 197 (1) (2008) 41–50.
[36] L. Xie, J. Ding, F. Ding, Gradient based iterative solutions for general linear matrix equations, Computers & Mathematics with Applications 58 (7) (2009) 1441–1448.
[37] F. Ding, Transformations between some special matrices, Computers & Mathematics with Applications 59 (8) (2010) 2676–2695.
[38] J. Ding, Y.J. Liu, F. Ding, Iterative solutions to matrix equations of form AiXBi = Fi, Computers & Mathematics with Applications 59 (11) (2010) 3500–3507.
[39] L. Xie, Y.J. Liu, H.Z. Yang, Gradient based and least squares based iterative algorithms for matrix equations $AXB + CX^{T}D = F$, Applied Mathematics and Computation 217 (5) (2010) 2191–2199.
[40] A.G. Wu, X.L. Zeng, G.R. Duan, W.J. Wu, Iterative solutions to the extended Sylvester-conjugate matrix equations, Applied Mathematics and Computation 217 (1) (2010) 130–142.
[41] M. Dehghan, M. Hajarian, An iterative algorithm for solving a pair of matrix equations $AYB = E$, $CYD = F$ over generalized centro-symmetric matrices, Computers & Mathematics with Applications 56 (12) (2008) 3246–3260.
[42] M. Dehghan, M. Hajarian, An iterative algorithm for the reflexive solutions of the generalized coupled Sylvester matrix equations and its optimal approximation, Applied Mathematics and Computation 202 (2) (2008) 571–588.
[43] F. Ding, T. Chen, Performance bounds of the forgetting factor least squares algorithm for time-varying systems with finite measurement data, IEEE Transactions on Circuits and Systems I: Regular Papers 52 (3) (2005) 555–566.
[44] F. Ding, P.X. Liu, G. Liu, Identification methods for Hammerstein nonlinear systems, Digital Signal Processing (2011) doi:10.1016/j.dsp.2010.06.006.
[45] D.Q. Wang, Y.Y. Chu, G.W. Yang, F. Ding, Auxiliary model-based recursive generalized least squares parameter estimation for Hammerstein OEAR systems, Mathematical and Computer Modelling 52 (1–2) (2010) 309–317.
[46] D.Q. Wang, F. Ding, Extended stochastic gradient identification algorithms for Hammerstein–Wiener ARMAX Systems, Computers & Mathematics with Applications 56 (12) (2008) 3157–3164.
[47] F. Ding, Y. Shi, T. Chen, Auxiliary model based least-squares identification methods for Hammerstein output-error systems, Systems & Control Letters 56 (5) (2007) 373–380.
[48] F. Ding, T. Chen, Z. Iwai, Adaptive digital control of Hammerstein nonlinear systems with limited output sampling, SIAM Journal on Control and Optimization 45 (6) (2006) 2257–2276.
[49] F. Ding, T. Chen, Combined parameter and output estimation of dual-rate systems using an auxiliary model, Automatica 40 (10) (2004) 1739–1748.
[50] F. Ding, T. Chen, Parameter estimation of dual-rate stochastic systems by using an output error method, IEEE Transactions on Automatic Control 50 (9) (2005) 1436–1441.
[51] F. Ding, T. Chen, Hierarchical identification of lifted state-space models for general dual-rate systems, IEEE Transactions on Circuits and Systems I: Regular Papers 52 (6) (2005) 1179–1187.
[52] F. Ding, P.X. Liu, H.Z. Yang, Parameter identification and intersample output estimation for dual-rate systems, IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 38 (4) (2008) 966–975.
[53] J. Ding, Y. Shi, H.G. Wang, F. Ding, A modified stochastic gradient based parameter estimation algorithm for dual-rate sampled-data systems, Digital Signal Processing 20 (4) (2010) 1238–1249.
[54] L.L. Han, J. Sheng, F. Ding, Y. Shi, Auxiliary models based recursive least squares identification for multirate multi-input systems, Mathematical and Computer Modelling 50 (7–8) (2009) 1100–1106.
[55] Y.J. Liu, L. Xie, F. Ding, An auxiliary model based recursive least squares parameter estimation algorithm for non-uniformly sampled multirate systems, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering 223 (4) (2009) 445–454.
[56] F. Ding, L. Qiu, T. Chen, Reconstruction of continuous-time systems from their non-uniformly sampled discrete-time systems, Automatica 45 (2) (2009) 324–332.
[57] F. Ding, G. Liu, X.P. Liu, Partially coupled stochastic gradient identification methods for non-uniformly sampled systems, IEEE Transactions on Automatic Control 55 (8) (2010) 1976–1981.
[58] Y. Shi, F. Ding, T. Chen, Multirate crosstalk identification in xDSL systems, IEEE Transactions on Communications 54 (10) (2006) 1878–1886.
[59] Y. Shi, F. Ding, T. Chen, 2-norm based recursive design of transmultiplexers with designable filter length, Circuits, Systems and Signal Processing 25 (4) (2006) 447–462.
[60] Y. Shi, H. Fang, Kalman filter based identification for systems with randomly missing measurements in a network environment, International Journal of Control 83 (3) (2010) 538–551.
[61] Y. Shi, H. Fang, M. Yan, Kalman filter based adaptive control for networked systems with unknown parameters and randomly missing outputs, International Journal of Robust and Nonlinear Control 19 (18) (2009) 1976–1992. (special issue on Control with Limited Information, Part II).
[62] B. Yu, Y. Shi, H. Huang, l-2 and l-infnity filtering for multirate systems using lifted models, Circuits, Systems, and Signal Processing 27 (5) (2008) 699–711.
[63] H. Fang, J. Wu, Y. Shi, Genetic adaptive state estimation with missing input/output data, Proceedings of the Institution of Mechanical Engineers, Part I, Journal of Systems and Control Engineering 224 (5) (2010) 611–617.