Distributed Scheduling of Wireless Communications for Voltage Control in Micro Smart Grid

Husheng Li and Zhu Han

Abstract—**Voltage control is an important task in micro grid, which adjusts the voltages of the distributed energy generators (DEGs). Wireless communication is applied to convey the information from the voltage sensors to the DEGs. Due to the interference among the wireless sensors, scheduling is necessary for the transmissions of sensors. A distributed scheduling algorithm is proposed within the framework of hybrid dynamical systems. The discrete state in the system dynamics is obtained according to the scheduled sensors. Then, the scheduling is formulated as the problem of optimizing the mode sequence of the system dynamics and is then solved using sub-gradient algorithm in a distributed manner. The performance is demonstrated by numerical simulations.**

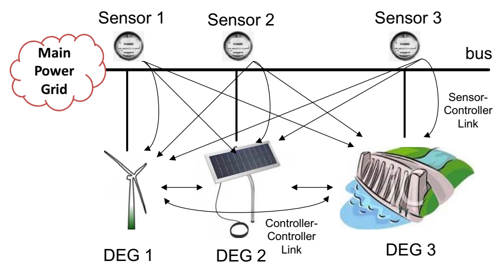

Fig. 1: An illustration of the voltage control in micro grid.

I. Introduction

In recent years, intensive studies have been paid to the development of smart grid, in which modern information technologies are applied to improve the reliability and efficiency [4]. In various technologies of smart grid, micro grid has attracted significant attentions [6]. In a micro grid, multiple distributed energy generators (DEGs), such as wind turbines and solar panels, can provide power to the users within the micro grid and are also connected to the main power grid. When there is a major damage in the main power grid, the micro grid can be disconnected from the main power grid, thus working in the island mode and avoiding the local blackout. Many testbeds of micro grid have been implemented; e.g., the Electric Reliability Technology Solutions (CERTS) Microgrid supported by the US Department of Energy [6].

An important task in the management of micro grid is the voltage regulation. At each DEG, an inverter converts the power at the DEG into AC electricity and regulates the voltage at the access point to the main bus by changing the voltage at the output of the inverter. There always exists randomness in the power grid; e.g., the arrival or leave of loads, thus making the voltages at different points deviate from the reference values. Hence, it is necessary to control the voltages at different DEGs by monitoring the voltages at the access points. A practical scheme for the voltage monitoring control is to adopt wireless technologies due to its flexibility and fast deployment; moreover, a wireless network can effectively cover the range of a micro grid (e.g., multiple square miles). As illustrated in Fig. 1, in which three DEGs are shown, the sensors send back the readings of voltages back to the DEGs for the control; the DEGs may also communicate with each other for coordination of voltage control actions[1]. The wireless network for the voltage control can be either centralized using a base station or decentralized using an ad hoc topology. Fig. 1 shows a decentralized one.

In this paper, *we study the traffic scheduling, a key issue in the communication network operation for the voltage control in micro grid, in the decentralized network structure.* The purpose is to efficiently schedule the wireless transmission in the MAC layer such that the micro grid can be reliably controlled. Note that the centralized case has been studied by us in [11], which employed the framework of hybrid system in which both continuous and discrete dynamics exist [10]. The key idea is that *the scheduling is equivalent to selecting the mode sequence of the corresponding hybrid system, thus converting the scheduling problem to the optimization of the mode sequence selection, which has been studied in the area of hybrid systems.* The decentralized scheduling is more complicated since the optimization has to be done in a distributed manner. To our best knowledge, there have not been any studies on the distributed scheduling for the communication network for networked control systems (also known as cyber physical systems). The framework proposed in this paper can also be used for the design of communication networks in many other networked control systems like robotic networks or unmanned aerial vehicular (UAV) networks.

Note that the design of communication systems for networked control has been studied in the seminal works by Liu [8] [9] and Xiao [15]. In [8] [9], the MAC layer of the wireless communication for networked control is optimized mainly using numerical methods. In [15], the impact of communication rate constraint is considered to be the quantization error; then the resource allocation for linear feedback control is formulated as an optimization problem such that the communication

H. Li is with the Department of Electrical Engineering and Computer Science, the University of Tennessee, Knoxville, TN (email: husheng@eecs.utk.edu). Z. Han is with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX (email: hanzhu22@gmail.com). This work was supported by the National Science Foundation under grants CCF-0830451 and ECCS-0901425.

[1]It has been demonstrated that the voltage control with the coordination among DEGs outperforms the one without coordination [7].

and control sub-systems are optimized jointly. In recent years, due to the emergence of study on cyber physical systems, the studies on communications for control are resurrecting; e.g., in [2], the IEEE 802.11 protocol is tuned for networked control systems. However, none of these studies proposed a unified framework for the design of communication systems in networked control. Despite some simplified assumptions, this paper provides a comprehensive framework for designing the communication for networked control, thus paving a way to the future studies and shortening the gap between the communities of control and telecommunication.

The remainder of the paper is organized as follows. The system models for the voltage in power grid and the communication network are given in Section II, which will be fit into the hybrid system framework in Section III. The scheduling strategy will be optimized in Section IV. Numerical results and conclusions are provided in Sections V and VI, respectively.

II. System Model

In this section, we introduce the system model. We first explain the model of power system dyanmics. Then, we introduce the model for communication system.

A. Power System Model

In this paper, we consider a micro grid with N DEGs and N sensors[2]. We denote by $\mathbf{x}$, an N-vector, the differences between the voltages measured at the sensors and the reference voltage. For simplicity, we consider only small perturbations which make the voltages slightly deviate from the reference values $\mathbf{v}_{ref}$. By linearizing around the reference values, we can write the system state dynamics as

$$\left\{ \begin{array}{ll} \mathbf{x}(t+1) & = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{n}(t), \\ \mathbf{y}(t) & = \mathbf{C}\mathbf{x}(t) + \mathbf{w}(t) \end{array} \right. , \tag{1}$$

where $\mathbf{x} \triangleq \mathbf{v} - \mathbf{v}_{ref}$, $\mathbf{v}$ is the vector of voltage values, $\mathbf{y}$ is the observation, $\mathbf{n}$ and $\mathbf{w}$ are noise. Note that $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are parameters of the system. The linear dynamics are valid for the small perturbation assumption. For larger perturbations, a PID controller is used in [7], which requires more complicated analysis and is thus beyond the scope of this paper. Note that the observation $\mathbf{y}$ is also equal to the system state $\mathbf{x}$; i.e., $\mathbf{C} = \mathbf{I}$, if the system state can be observed perfectly.

For the system dynamics, we have the following assumptions:

- We assume that the matrices $\mathbf{A}$ and $\mathbf{B}$, which are mainly determined by the power grid topology, electricity load and transmission lines, are both known to the DEGs, sensors and the communication network. In practice, these matrices may be unknown in advance. We can obtain the parameters via system identification. If there is a significant change in the power grid; e.g., a large load is tripped, the matrices will be unknown to the system; in this case, our study can provide a performance bound for the practical control.
- The matrices $\mathbf{A}$ and $\mathbf{B}$ are diagonal; i.e., the the system state evolution and control are completely decoupled. This can significantly reduce the complexity of analysis and is reasonable if the DEG access points are far away. It also provides a starting point for studying the complicated scenario in which the system dynamics and controls are significantly coupled[3].
- We consider a linear feedback control law by assuming

$$\mathbf{u}(t) = \mathbf{L}\hat{\mathbf{x}}(t), \tag{2}$$

  where $\mathbf{L}$ is a constant feedback gain matrix and $\hat{\mathbf{x}}(t)$ is the estimation of the current system state deviation. Although it may achieve a better performance to employ a time varying $\mathbf{L}$, we fix it to simplify the analysis.
- For the system state estimation $\hat{\mathbf{x}}(t)$, we use the output of the standard Kalman filter.

B. Communication System Model

The voltage values will be packed into data packets that may also contain other information like frequency measurements, currents, temperature, power flow, wind speed or even video. We assume that the voltage information is the most important one and the scheduling is based on the voltage. For simplicity, we consider only a unicast traffic in the communication network for the voltage control; i.e., each sensor sends its data to only one DEG. Moreover, we assume that there is no communication among the DEGs although it has been demonstrated that the inter-DEG communication for control coordination is beneficial [7]. In the future, the assumptions can be relaxed and the system is thus extended to the following two cases:

- Multicast: each sensor multicasts its observation to multiple DEGs; in this case, network coding may be useful for reducing the required bandwidth.
- Inter-DEG communication: the DEGs can communicate with each other to coordinate the control.

The two advanced scenarios are too complicated for the current study. Hence, we stick to the simple model and expect to provide insights for the more practical case.

We assume that all communications are single-hop. The sensors and DEGs form a network with $2N$ nodes that can be represented by a bipartite graph. We assume that a single communication channel is used for all nodes and two nearby transmissions will cause a collision. An interference map can be used to represent the possible sets of links that cannot transmit simultaneously. We denote by $i \sim j$ if the transmissions of sensors i and j conflict with each other.

The complicated case of multihop and multichannel communications will be left to our future study. Moreover, we assume that, once scheduled for transmission, the packet containing the voltage information will be received correctly. The case with packet losses can be addressed using the techniques in [3]. For simplicity, we also ignore the quantization error. We assume that the sensors are perfectly synchronized in time, which can be achieved by the GPS equipped by each sensor.

[2]The number of DEGs and the number of sensors are not necessarily the same. For simplicity, we assume that they are equal to each other. The proposed work can be easily extended to the generic case.

[3]Note that our work in [11] is focused on coupled systems but assumes a centralized control using a base station.

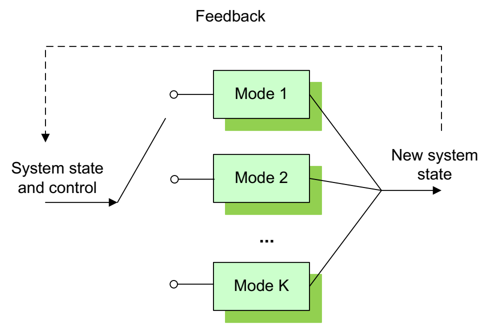

Fig. 2: An illustration of hybrid systems.

III. Hybrid System Framework

In this section, we fit the voltage control with a wireless communication network into the framework of hybrid systems [10]. We first introduce the theory of hybrid system and then formulate the hybrid system in the voltage control task. Particularly, we focus on the special case of switching linear system [12].

A. Hybrid System: Switching Linear System

A generic hybrid system is illustrated in Fig. 2. For simplicity, we consider only a special case of hybrid systems; i.e., a switching linear system that contains both continuous system state $\mathbf{x}(t)$ and discrete system state $\mathbf{k}(t)$. The system dynamics can be written as

$$\left\{ \begin{array}{l} \mathbf{x}(t+1) = \mathbf{A}_{k_1(t)}\mathbf{x}(t) + \mathbf{B}_{k_2(t)}\mathbf{u}(t) + \mathbf{n}(t) \\ \mathbf{y}(t) = \mathbf{C}_{k_3(t)}\mathbf{x}(t) + \mathbf{w}(t) \end{array} \right. ,$$

where $\mathbf{n}$ and $\mathbf{w}$ are noise, $\mathbf{u}$ is the control action, $\mathbf{y}$ is the vector of observations. There are finite selections of the system matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, and the 3-tuple $\mathbf{k} = (k_1(t), k_2(t), k_3(t))$ specifies the mode of the system dynamics at time t. Due to its difficulty, the stability and control strategy including controlling both the control action $\mathbf{u}$ and mode $\mathbf{k}$ are still open problems; e.g., [13] studied the optimization of both states by decomposing the optimization into master and slave problems.

B. Hybrid System for Voltage Control

In this subsection, we fit the task of voltage control, together with the communication system, into the framework of hybrid systems. We will explain the observation matrix, the cost function, the system state estimation and the controller synthesis in this subsection.

1) Observation Matrix: In the context of voltage control, the matrices $\mathbf{A}$ and $\mathbf{B}$ do not change with time. The only time-varying parameter in the system dynamics (1) is the observation matrix $\mathbf{C}$. Suppose that, at time slot t, sensors n_1, ..., n_m are allowed to transmit. We denote by $\Omega(t) = \{n_1, ..., n_m\}$ these scheduled sensors. Due to the assumption of perfect quantization and transmission, we have $y_{n_i}(t) = x_{n_i}(t)$, $i = 1, ..., m$. For other sensors, we have $y_{n_j} = 0$, $\forall j \notin \{n_1, ..., n_m\}$. Hence, for time slot t, we have

$$\mathbf{C}_{ii} = \left\{ \begin{array}{ll} 0, & \text{if } i \notin \Omega(t) \\ 1, & \text{if } i \in \Omega(t) \end{array} \right. . \quad (3)$$

Note that the off-diagonal elements of $\mathbf{C}$ are all zero. Essentially, the scheduling of communication network is to control the diagonal elements of the observation matrix $\mathbf{C}$.

2) Cost Function: We define the cost function of the hybrid system for voltage control as the weighted sum of the square deviation of voltages; i.e.,

$$J = \sum_{t=0}^{\infty} \beta^t E\left(\mathbf{x}^T(t)\mathbf{\Sigma}\mathbf{x}(t)\right), \quad (4)$$

where β is a discount factor, $\mathbf{\Sigma}$ is a diagonal matrix. If all voltages play the same role, $\mathbf{\Sigma}$ should be an identity matrix.

3) System State Estimation: We use the Kalman filtering to estimate the system state which will be used as the input to the controller. The estimation is given by

$$\mathbf{x}(t|t) = \mathbf{x}(t|t-1) + \mathbf{K}(t)\left[\mathbf{y} - \mathbf{C}_{k_t}\mathbf{x}(t|t-1)\right], \quad (5)$$

where

$$\mathbf{x}(t+1|t) = \mathbf{A}\mathbf{x}(t|t), \quad (6)$$

and

$$\mathbf{K}(t) = \mathbf{\Sigma}(t|t-1)\left[\mathbf{C}_{k_t}\mathbf{\Sigma}(t|t-1)\left(\mathbf{C}_{k_t}\right)^T + \sigma_n^2\mathbf{I}\right]^{-1}, \quad (7)$$

and

$$\mathbf{\Sigma}(t|t) = \mathbf{\Sigma}(t|t-1) - \mathbf{K}_t\mathbf{C}_{k_t}\mathbf{\Sigma}(t|t-1), \quad (8)$$

where (here $\mathbf{Q}$ is the covariance matrix of noise)

$$\mathbf{\Sigma}(t+1|t) = \mathbf{A}\mathbf{\Sigma}(t|t)\mathbf{A}^T + \mathbf{B}\mathbf{Q}\mathbf{B}^T. \quad (9)$$

4) Controller Synthesis: We use the linear quadratic regulator (LQR) controller based on the system state estimation, where the feedback gain matrix is given by

$$\mathbf{L} = \left(\mathbf{B}^T\mathbf{S}\mathbf{B} + \mathbf{P}\right)^{-1}\mathbf{B}^T\mathbf{S}\mathbf{A}. \quad (10)$$

The matrix $\mathbf{S}$ is determined by the algebraic Riccati Equation, which is given by

$$\mathbf{S} = \mathbf{A}^T\left[\mathbf{S} - \mathbf{S}\mathbf{B}\left(\mathbf{B}^T\mathbf{S}\mathbf{B} + \mathbf{P}\right)^{-1}\mathbf{B}^T\mathbf{S}\right]\mathbf{A} + \mathbf{Q}. \quad (11)$$

IV. Distributed Scheduling

Due to the interference and the lack of a centralized controller, the voltage sensors need to schedule their transmissions in a distributed manner. We will first formulate the centralized scheduling problem and then decentralize it.

A. Centralized Scheduling

1) Formulation: In a centralized case, the scheduling can be cast into the following discrete optimization problem:

$$\begin{array}{lll} & \min_{\Omega} J(\Omega) & \\ s.t. & i \nsim j, & \text{if } i, j \in \Omega, \end{array} \quad (12)$$

where the action is the set of scheduled sensors, which is subject to the constraint of interference. We assume that there are totally M constraints due to the wireless interference.

We can convert the problem into an integer programming by defining a binary vector $\mathbf{z}$ such that

$$z_i = \begin{cases} 1 & , \quad \text{sensor } i \text{ scheduled} \\ 0 & , \quad \text{otherwise} \end{cases} . \tag{13}$$

Then, the problem is given by

$$\begin{aligned} & \min_{\mathbf{z}} J(\mathbf{z}) \\ s.t. \quad & z_i + z_j \leq 1, \qquad \text{if } i \sim j \\ & z_i \in \{0,1\}, \qquad \forall i. \end{aligned} \tag{14}$$

Equivalently, the constraint can be written as $\mathbf{Dz} \leq \mathbf{1}$, where the $M \times N$ matrix $\mathbf{D}$ is the connection matrix of the interference graph.

2) Challenges: Even for the centralized case, there are still two difficulties for the scheduling:

- There is no explicit expression for the function $J(\mathbf{z})$. On one hand, it is hard to obtain the explicit expression even when all actions are given. On the other hand, the future actions are still uncertain due to the uncertain future system state.
- The binary integer constraints may make the optimization an NP-hard one.

3) Monte-Carlo Approach: To address the first difficulty, we use Monte-Carlo simulations to approximate the cost function $J(\mathbf{z})$; i.e., generating the random future system state and the corresponding actions. A challenge for the Monte-Carlo computation of the objective function is the computation for the future cost that concerns the computation of the future actions. For example, when the current time is t and the assumed action is $\mathbf{z}(t)$, there could be an uncountable possibilities of $\mathbf{x}(t+1)$ due to the random noise in the dynamics; to compute the system state $\mathbf{x}(t+2)$ for the objective function, we need to do the optimization for $\mathbf{z}(t+2)$ for each $\mathbf{x}(t+1)$, which incurs significant computations even if $\mathbf{x}(t+1)$ can be discretized; for further computing the future system states $\mathbf{x}(t+3)$, $\mathbf{x}(t+4)$..., the computation will be prohibitively difficult. Hence, it is difficult to consider a precise evaluation of the cost function J. For simplifying the analysis, we consider a myopic strategy; i.e., considering only one step further and ignoring the cost incurred by the future system states in time slots beyond $t+1$. As will be seen, such a myopic strategy will result in a good performance. We will study the performance loss incurred by the myopic strategy in the future.

Given the action $\mathbf{z}$ and the current system state $\mathbf{x}$, the expected objective function of the myopic strategy can be written as

$$E\left[J(\mathbf{z})\right] = \left[\mathbf{Ax} + \mathbf{Bu}(\mathbf{z})\right]^T \mathbf{\Sigma} \left[\mathbf{Ax} + \mathbf{Bu}(\mathbf{z})\right] + \text{trace}(\mathbf{\Sigma}), \tag{15}$$

where $\mathbf{u}$ is a function of $\mathbf{z}$ and the future cost beyond the next time slot is ignored due to the myopic policy. This can be easily computed.

4) Lagrangian Relaxation: To address the second difficulty, we plan to use the Lagrangian relaxation approach [14] in integer programming. Although there are many other effective approaches for integer programming like branch and bound algorithm, cutting plane algorithm and column generation algorithm [14], we select the Lagrangian relaxation approach due to its capability of decentralization. To that end, we define $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_M)$ (recall that M is the number of constraints; i.e., the number of edges in the interference graph), where $\lambda_i \geq 0$, $\forall i$, and consider the following Lagrangian relaxation problem:

$$\begin{aligned} & \min_{\mathbf{z}} J(\mathbf{z}) + \boldsymbol{\lambda}^T \left(\mathbf{Dz} - \mathbf{1}\right) \\ & \mathbf{z} \in \{0,1\}^M. \end{aligned} \tag{16}$$

The following proposition discloses the properties of the Lagrangian relaxation problem in (16). The proof is omitted since it is similar to those of Propositions 10.1 and 10.2 in [14]; the only difference is that the objective function in this paper is nonlinear.

Proposition 4.1: (A) The Lagrangian relaxation problem in (16) is a relaxation of the integer programming in (14).

(B) If a $\boldsymbol{\lambda} \geq 0$ satisfies (here $\mathbf{z}(\boldsymbol{\lambda})$) is a function of $\boldsymbol{\lambda}$)

- $\mathbf{z}(\boldsymbol{\lambda})$ is optimal for the Lagrangian relaxation in (16),
- $\mathbf{Dz}(\boldsymbol{\lambda}) \leq \mathbf{1}$,
- $\left(\mathbf{Dz}(\boldsymbol{\lambda})\right)_i = 1$, if $u_i > 0$,

then $\mathbf{z}\boldsymbol{\lambda}$ is also optimal for the integer programming problem in (14).

We denote by $\phi(\boldsymbol{\lambda})$ the optimal value of the optimization problem in (16), as a function of $\boldsymbol{\lambda}$. Then, we can maximize $\phi(\boldsymbol{\lambda})$ in order to obtain the optimal (or near optimal) solution $\mathbf{x}$. In the centralized configuration, this can be achieved by the subgradient algorithm, in which the value of $\boldsymbol{\lambda}$ is updated by finding the subgradient [14], i.e.,

$$\boldsymbol{\lambda}^{k+1} = \max\left\{0, \boldsymbol{\lambda}^k - \epsilon_k \boldsymbol{\xi}^k\right\}, \tag{17}$$

where $\boldsymbol{\xi}^k$ is the subgradient and ϵ_k is the step factor. It can be verify that $\boldsymbol{\xi}^k = \mathbf{1} - \mathbf{Dz}(\boldsymbol{\lambda}^k)$ is a subgradient. The details are omitted and can be found in [14].

B. Distributed Scheduling

Now we study the distributed scheduling based on the discussion for the centralized one. The distributed scheduling faces the following challenges:

- In contrast to the centralized one, each sensor only knows its own measurement and does not know the situations of other sensors. Hence, it does not know the value of function $J(\mathbf{z})$ and cannot accomplish the optimization in (16).
- In the distributed scheduling for pure data communication networks; e.g., the back pressure scheduling algorithm that stabilizes the queuing dynamics in wireless networks, certain information exchange needs to be carried out for the scheduling, thus requiring a preamble before the data transmission. For example, the queue length information or certain scheduling metrics will be exchanged among neighbors. However, such an information exchange is not available in the context of voltage control due to the stringent realtime requirement and limited bandwidth. Hence, each scheduling will be completed without any explicit information exchange among the sensors.

1) Decomposition: To address the above challenges, we first decompose the problem in (16) into a distributed one. Thanks to the assumption of decoupled dynamics of the voltage evolution, (16) can be written as

$$\min_{\mathbf{z}} \sum_{n=1}^{N} \left[J_n(z_n) + w_n(\boldsymbol{\lambda}) z_n\right]$$
$$s.t. \qquad \mathbf{z} \in \{0,1\}^M, \tag{18}$$

where $J_n = \sum_{t=0}^{\infty} \beta^t E\left(x_n^2 \boldsymbol{\Sigma}_{nn}\right)$ and $w_n(\boldsymbol{\lambda})$ is the coefficient for x_n in the linear form $\boldsymbol{\lambda}^T(\mathbf{Dz} - \mathbf{1})$, as a function of $\boldsymbol{\lambda}$.

Due to the above decomposition of the objective function, we can decompose the optimization problem into N subproblems:

$$\min_{z_n} \left[J_n(z_n) + w_n(\boldsymbol{\lambda}) z_n\right]$$
$$s.t. \qquad z_n \in \{0,1\}, \tag{19}$$

for $n = 1, ..., N$. Then, given a $\boldsymbol{\lambda}$ and the corresponding w_n, each sensor can optimize z_n using any integer optimization method.

Again, we will use the myopic policy to compute the objective function J_n, which is given by

$$J_n(z_n) = (\mathbf{A}_{nn} x_n + \mathbf{B}_{nn} u_n(z_n))^2 \Sigma_{nn}. \tag{20}$$

Sensor n can simply evaluate $z_n = 0, 1$ and then obtain the optimal decision on z_n.

2) Adjustment of $\boldsymbol{\lambda}$: As we have explained, the optimization problem can be decomposed into distributed problems given the Lagrange factor $\boldsymbol{\lambda}$. Actually, such a decomposition is very similar to that of the Network Utility Maximization (NUM) [5]. However, in the theory of NUM, the Lagrange factor, namely the price, is updated according to the current network congestion situation, thus requiring information exchange among the agents or the price broadcast from a center. As we have assumed, there is no information exchange among the sensors and there is no control center, thus preventing an explicit mechanism for adjusting the Lagrange factor $\boldsymbol{\lambda}$.

One simple approach is to fix a constant $\boldsymbol{\lambda}$. However, to make the scheduling more flexible, we borrow the idea from congestion control like TCP in which there is no explicit control center adjusting the Lagrange factor or price. We assume that each sensor is able to detect the collision caused by the simultaneous transmission of multiple sensors. Then, the sensor adjust the coefficient w_n directly, instead of $\boldsymbol{\lambda}$: when there is no collision, $w(n)$ is decreased slightly; when there is a collision, w_n is increased significantly.

In our simulation, we adopt the following rule for updating w_n:

$$w_n(t+1) = \begin{cases} \max[0, w_n(t) - \delta w], \text{ if no collision} \\ 2w_n(t+1), \text{if collision} \end{cases}, \tag{21}$$

where δw is a constant step of the weight decrease.

Intuitively, w_n stands for the penalty due to the congestion incurred by the collisions. The sensor decides the transmission by considering both the importance of the information, namely J_n, and the penalty priced by w_n. With the updating rule in (21), the sensors can adaptively adjust the balance between the information importance and the collision. The algorithm is summarized in Procedure 1.

Procedure 1 Procedure of Distributed Scheduling for Voltage Control

```
1: Set initial coefficients {w_n}_{n=1,...,N}.
2: for Each time slot do
3:    for Each sensor do
4:       Compute the cost function J_n
5:       Optimize J_n(z_n) + w_n z_n
6:       Decide the transmission
7:    end for
8:    Data transmission
9:    for Each sensor do
10:      Adjust w_n according to (21)
11:   end for
12: end for
```

V. Numerical Results

In this section, we use numerical simulations to demonstrate the performance of the proposed distributed scheduling algorithm.

A. Simulation Setup

We consider a micro grid with three DEGs. We assume that sensors 1 and 2 cannot transmit simultaneously while sensors 2 and 3 cannot transmit simultaneously. We assume that the voltage dynamics are given by

$$A = \begin{pmatrix} 1.03 & 0 & 0 \\ 0 & 1.02 & 0 \\ 0 & 0 & 1.05 \end{pmatrix}, \tag{22}$$

$$B = \begin{pmatrix} 0.6 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.8 \end{pmatrix}. \tag{23}$$

Note that the matrix $\mathbf{A}$ is the same as that in our previous study in which the scheduling is centralized [11] while the matrix $\mathbf{B}$ is different in the off-diagonal elements (in this paper, all off-diagonal elements are set to zero such that the system is decoupled). We use the identity matrix for the cost function. For performance comparison, we also consider the simple round-robin scheduling algorithm as the baseline; i.e., the sensors transmit in the order of 1/3, 2, 1/3, 2,... (recall that sensors 1 and 3 can transmit simultaneously).

B. Simulation Results

In the simulation, we assume that the voltage deviations of the three sensors are increased to 30V, 10V and 20V at time 0, respectively. Then, the DEGs will take action according to the sensor reports to control the voltage deviation back to 0.

The voltage evolutions of both scheduling algorithms are shown in Fig. 3. We observe that both scheduling algorithms can drive the voltage to converge to the reference value. The accumulated cost, as a function of time, is shown in Fig. 4. We observe that the proposed scheduling algorithm can significantly reduce the cost due to the voltage deviation. The main reason of the cost reduction is that the voltage at sensor 3 can be decreased faster when the proposed scheduling algorithm is applied, as demonstrated in Fig. 5.

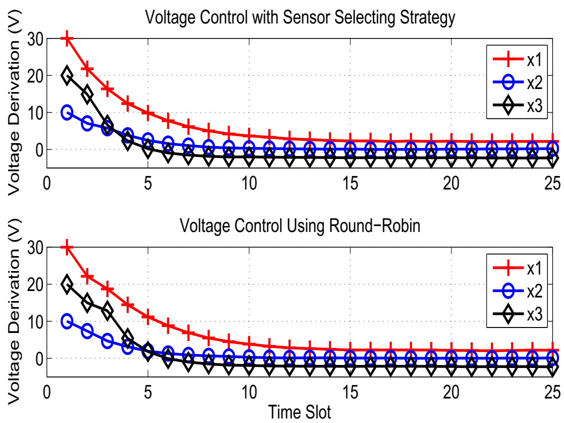

Fig. 3: Voltage evolution with time.

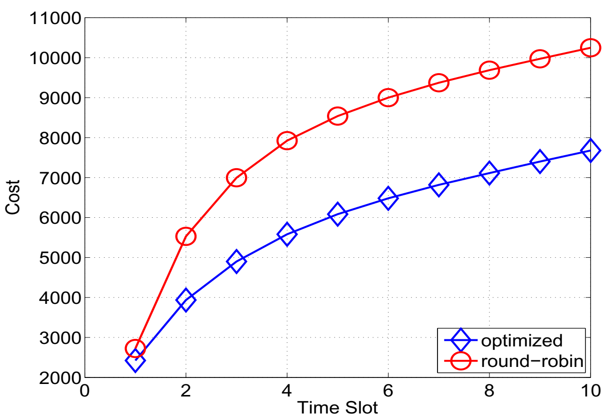

Fig. 4: Accumulated cost with time.

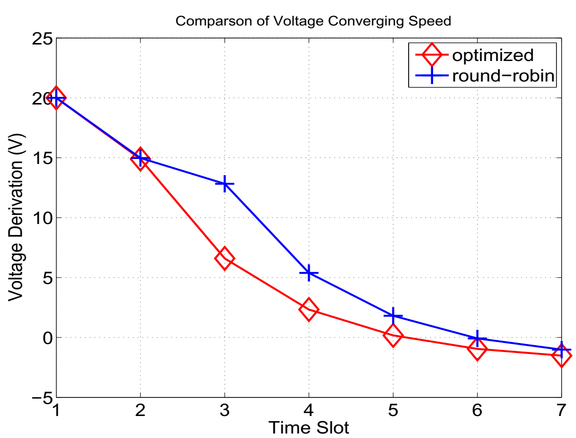

Fig. 5: Comparison of voltage at sensor 1.

VI. Conclusions

We have studied the distributed scheduling for the voltage control in micro grid. The centralized scheduling has been formulated as an integer programming problem, for which we have proposed to use the Lagrange relaxation approach to solve. For the distributed case, we have decomposed the integer programming problem into sub-problems which can be solved using local observations, based on a given Lagrange factor and the assumption of decoupled dynamics of voltages. We have also proposed a simple approach to adjust the Lagrange factor without explicit information exchange among the sensors, by borrowing the idea of TCP based congestion control. Numerical simulations on a three-DEG system have demonstrated the performance of the proposed scheduling algorithm.

Our future work includes the following aspects: (a) Consider more generic communication networks such as multihop and multicast ones, as well as the inter-DEG communications; (b) Consider more generic micro grid in which the dynamics of different voltages are coupled; (c) Implement the communication network in a real micro grid.

References

[1] V. Ajjarapu, *Computational Techniques for Voltage Stability Assessment and Control* , Springer, 2006.

[2] J. Bai, E. P. Eyisi, Y. Xue and X. D. Koutsoukos, "Dynamic tuning retransmission limit of IEEE 802.11 MAC protocol for networked control systems," in *Proc. of the 1st International Workshop on Cyber-Physical Networking Systems (CPNS)*, 2011.

[3] J. P. Hespanha, P. Naghshtabrizi and Y. Xu, "A survey of recent results in NCS," *Proceedings of IEEE*, Vol. 95, pp. 138–162, Jan. 2007.

[4] ISO New England Inc., *Overview of the Smart Grid: Policies, Initiatives and Needs*, Feb. 17, 2009.

[5] F. P. Kelly, A. Maulloo and D. Tan, "Rate control for communication networks: Shadow prices, proportional fairness and stability," *J. Operation Res. Soc.*, vol.49, no.3, pp. 237–252, 1998.

[6] J. D. Kueck, R. H. Staunton, S. D. Labinov and B. J. Kirby, "Microgrid energy management system," Technical Report ORNL/TM-2002/242, Oak Ridge National Lab, 2002.

[7] H. Li, F. Li, Y. Xu, D. T. Rizy and J. D. Kueck, "Adaptive voltage control with distributed energy resources: Algorithm, theoretical analysis, simulation, and field test verification," *IEEE Trans. on Power Systems*, vol. 25, no. 3, Aug., 2010.

[8] X. Liu and A. Goldsmith, "Wireless medium access control in distributed control systems," in *Proc. of Allerton Annul Conference on Communications, Control and Computing*, 2003.

[9] X. Liu and A. Goldsmith, "Wireless network design for distributed contorl," in *Proc. of IEEE Conference on Decision and Control (CDC)*, 2004.

[10] J. Lunze and F. L. Lararrigue, *Handbook of Hybrid Systems Control: Theory, Tools and Applications*, Cambridge Univ. Press, 2009.

[11] R. Mao and H. Li, "Nobody but you: Sensor selection for voltage regulation in smart grid," submitted to *IEEE Conference on Global Communications (GLobecom)*, 2011.

[12] M. Mariton, *Jump Linear Systems in Automatic Control*, Marcel Dekker Inc., 1990.

[13] C. Seatzu, D. Corona, A. Giua and A. Bemporad, "Optimal control of continous-time switched affine systems," *IEEE Trans. on Automatic Control*, vol.51, pp.726–741, May 2006.

[14] L. A. Wolsey, *Integer Programming*, Wiley, 1998.

[15] L. Xiao, M. Johansson, H. Hindi, S. Boyd and A. Goldsmith, "Joint optimization of communication rates and linear systems," *IEEE Trans. Automat. Contr.*, vol.48, pp.148–153, Jan. 2003.