Automatica 48 (2012) 2710–2716

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A closed-loop data based test for robust performance improvement in iterative identification and control redesigns[☆]

Sourav Patra[a,1], Alexander Lanzon[b,2]

[a] Electrical Engineering Department, Indian Institute of Technology Kharagpur, 721302, India

[b] Control Systems Centre, School of Electrical and Electronic Engineering, University of Manchester, Manchester, M13 9PL, UK

ARTICLE INFO

Article history:
Received 11 March 2010
Received in revised form
1 February 2012
Accepted 10 June 2012
Available online 20 July 2012

Keywords:
Iterative identification and robust control
Robust stability margin
Robust performance
Positive-real

ABSTRACT

In robust iterative identification and control redesign techniques, a stabilizing controller connected in a closed loop is normally replaced by an alternative attractive stabilizing controller to improve robustness and performance of the closed-loop system. In this paper, novel test methods are proposed to check whether a new stabilizing controller improves performance or not when the existing controller is replaced by this new controller in the closed loop. The proposed tests are based on closed-loop data and no plant model, and can be used for both the SISO and MIMO linear time-invariant systems. For the proposed tests, the plant dynamics is assumed to be unknown whereas the existing and new controller transfer function matrices are known to the designer. These assumptions are common in iterative identification and control redesign techniques. The performance improvement test methods proposed in this paper build on the experimental set-up proposed in Dehghani, Lecchini, Lanzon, and Anderson (2009) which was used to only check whether controllers ensure internal stability of a feedback interconnection or not. In this paper, new test methods are proposed to ascertain robust performance improvement that cannot be obtained from test results of Dehghani et al. (2009). A numerical example is illustrated to show effectiveness of the proposed test methods.

1. Introduction

In iterative identification and robust control redesign techniques, a control systems engineer starts to identify the plant model based on the closed-loop data to design a more attractive controller so that the robustness of the closed-loop system is improved while the existing known controller is replaced by the new designed controller (Date & Lanzon, 2004; Gevers, 2000; Gevers, Bombois, Codrons, Scorletti, & Anderson, 2003; Hjalmarsson, Gevers, Gunnarson, & Lequin, 1998; Schrama, 1992). After inserting the new controller in the closed-loop system, the identification and controller redesign methods are repeated and it progresses iteratively until a satisfactory level of performance is achieved (Bitmead, 1993; Gevers, 2000, 2002; Schrama, 1992; Schrama & Van Den Hof, 1992). However, appreciating that any identified model will not be an exact representation of the actual physical plant present in the loop, this technique must always look for a 'safe' controller change which typically provides only bounds of maximum allowable robust performance degradation (e.g. based on some metric quantity the distance between the identified plant and the unknown physical plant) and consequently robust performance improvement may not be always possible to guarantee a priori (Anderson, 2004; Anderson & Gevers, 1998; Baldi, Battistelli, Mosca, & Tesi, 2010; Bitmead, 1993; Dehghani, Lanzon, & Anderson, 2004; Dehghani, Lecchini, Lanzon, & Anderson, 2009; Gevers, 2002; Lanzon, Leccchini, Dehghani, & Anderson, 2006; Lecchini, Lanzon, & Anderson, 2006; Manuelli, Cheong, Mosca, & Safonov, 2007; Schrama, 1992). It is assumed that the currently active controller internally stabilizes the existing closed loop. The plant model, identified in present iteration from the available closed-loop data, is hence close to the actual system in some sense, for example as measured by a ν-gap, as the existing known controller simultaneously stabilizes the actual unknown plant as well as the identified plant model. In the next step, an attractive new stabilizing controller, designed based on the identified plant model from the previous iteration, is to be inserted into the closed loop. It too must ensure internal stability with the actual plant. In Dehghani et al. (2009), test methods based on the existing closed-loop data are given to examine the potential of the attractive new controller to stabilize the actual plant before inserting into the closed loop. Dehghani et al. (2009) reduces the probability that a destabilizing

[☆] The material in this paper was partially presented at the 18th IFAC World Congress, August 28–September 2, 2011, Milan, Italy. This paper was recommended for publication in revised form by Associate Editor Gang Tao under the direction of Editor Miroslav Krstic.

E-mail addresses: sourav@ee.iitkgp.ernet.in (S. Patra), alexander.lanzon@manchester.ac.uk (A. Lanzon).

[1] Tel.: +91 3222 283030; fax: +91 3222 282261.

[2] Tel.: +44 161 3068722.

0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.06.087

controller will be switched into the loop which is undesirable in 'safe' iterative identification and control redesign techniques. For 'safe control', hence, it is always very important to check that the newly designed controller that seems to be attractive before inserting into the closed-loop system is guaranteed to at least stabilize the unknown plant (Anderson, 2004; De Callafon & Van Den Hof, 1997; Hildebrand, Lecchini, Solari, & Gevers, 2005; Hjalmarsson et al., 1998; Kammer, Bitmead, & Bartlett, 2000). Many data-based internal stability tests in the literature are based either on the parametric identification of the full order model of the current closed-loop system or on the full estimation of frequency bounds of magnitude of the current closed-loop transfer functions. In Baldi et al. (2010) and Manuelli et al. (2007), the data-driven test functions are used for choosing an appropriate controller in unfalsified adaptive control. In Dehghani et al. (2009), an alternative set of experiments was proposed to test internal stability of an apparently attractive controller based on data-only experiments which do not require the full frequency spectrum which prevent the possibility of inserting a destabilizing controller in the closed-loop system.

However, although ensuring internal stability of a newly designed controller on the unknown physical plant is a necessary prerequisite to an iterative identification and control redesign technique, it is not sufficient as it is important to ensure monotonic robust performance improvement when the designer has one or a set of attractive stabilizing controllers at hand. In this scenario, although the available controllers are all stabilizing, the following is an important question: which of these stabilizing controllers will improve performance when the existing controller is replaced by the newly chosen stabilizing controller? The present paper gives an answer to this question by proposing novel test methods based on closed-loop data. In this paper, the frequency response is considered which is obtained from input–output frequency-domain data. The swept sine test can also give the frequency response, however, it is practically impossible to perform a swept sine test at all frequencies. Hence, for sufficiently accurate frequency response, it is assumed that the time duration and the sampling rate are sufficiently large. It is also assumed that the physical plant is linear time-invariant and is unknown to the designer, whereas all controllers are assumed to be known. These assumptions are common in iterative identification and control redesign techniques. As the final objective of the design is to achieve the highest level of robust performance of the closed-loop system, it is important to insert a new controller that improves the monotonically robust performance of the closed-loop system and to this end, the proposed tests of this paper will enable this objective.

The proposed tests build on the experimental set-up proposed in Dehghani et al. (2009) where the tested controller is implemented in coprime factorization form. This distinctive implementation of the experimental set-up always gives a stable input–output map for the experiments even if the new controller is destabilizing. In this paper, we use the same experimental set-up as in Dehghani et al. (2009) to propose new additional test methods to ascertain robust performance improvement of the closed-loop system that cannot be obtained from the tests in Dehghani et al. (2009).

2. Notations and preliminaries

Let $\mathcal{R}$ denote the set of all real rational transfer function matrices and $\mathcal{RH}_\infty^{m\times n}$ be the set of all real rational stable transfer function matrices with m rows and n columns. Let a transfer function matrix $G \in \mathcal{R}$, then $\mathcal{L}_2$-adjoint system $G^*(s)$ denotes $G(-s)^T$. Let $\mathbb{R}$ and $\mathbb{C}$ denote the fields of real and complex numbers respectively. Also let $\mathbb{C}_-$ and $\bar{\mathbb{C}}_-$, respectively, denote the open and closed left-half planes. Let A^* denote the complex conjugate transpose of matrix A. Let $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$, respectively, denote the largest and the smallest singular value of matrix A. Let $\|P\|_\infty$ denote the $\mathcal{H}_\infty$-norm of $P \in \mathcal{RH}_\infty$. The number **wno**(.) indicates the winding number of a scalar transfer function evaluated on a standard D-contour indented to the right around any imaginary axis poles (Vinnicombe, 2000). The nearest integer function **nint**[.] returns the integer closest to [.] with the additional rule that half-integers are always rounded to even numbers. Let $\mathbb{X}$ be an inner product space, then the inner product of $\mathbb{X}$ is denoted by $\langle ., . \rangle_{\mathbb{X}} : \mathbb{X} \times \mathbb{X} \to \mathbb{R}$.

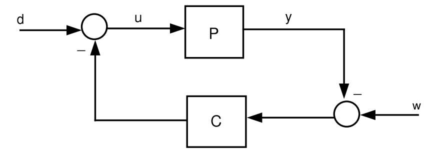

Fig. 1. Feedback interconnection of systems.

Consider the standard feedback interconnection of systems as shown in Fig. 1 where $P \in \mathcal{R}^{n\times m}$ and $C \in \mathcal{R}^{m\times n}$. From the exogenous input vector $\begin{bmatrix}\omega^T & d^T\end{bmatrix}^T \in \mathbb{C}^{n+m}$ to $\begin{bmatrix}y^T & u^T\end{bmatrix}^T \in \mathbb{C}^{n+m}$, the transfer function matrix is $H(P, C) = \begin{bmatrix}P\\ I\end{bmatrix}(I - CP)^{-1}\begin{bmatrix}-C & I\end{bmatrix}$.

Definition 1 (*Vinnicombe, 2000; Zhou, Doyle, & Glover, 1996*)**.** The interconnection $[P, C]$ as depicted in Fig. 1 is well-posed if $H(P, C)$ exists, and furthermore $[P, C]$ is said to be internally stable if it is well-posed and $H(P, C) \in \mathcal{RH}_\infty^{(n+m)\times(n+m)}$.

Note that, in four-block uncertainty structure the transfer function matrix from output to input of the uncertainty block is also depicted by $H(P, C)$ (Vinnicombe, 2000), and inverse of infinity norm of this transfer function gives the robust stability margin of the closed-loop system. This is also a measure for robust performance which will be explained later in Section 3. For simplicity's sake, in this paper the dimension of the transfer function matrix is not mentioned explicitly.

Definition 2. The ordered pair $\{N, M\}$ where $N, M \in \mathcal{RH}_\infty$, is a normalized right-coprime factorization (rcf) of $P \in \mathcal{R}$ if M is invertible in $\mathcal{R}$, $P = NM^{-1}$, and N and M are right-coprime over $\mathcal{RH}_\infty$ and $M^*M + N^*N = I$.

Definition 3. The ordered pair $\{\tilde{U}, \tilde{V}\}$ where $\tilde{U}, \tilde{V} \in \mathcal{RH}_\infty$, is a normalized left-coprime factorization (lcf) of $C \in \mathcal{R}$ if $\tilde{V}$ is invertible in $\mathcal{R}$, $C = \tilde{V}^{-1}\tilde{U}$, and $\tilde{U}$ and $\tilde{V}$ are left-coprime over $\mathcal{RH}_\infty$ and $\tilde{V}\tilde{V}^* + \tilde{U}\tilde{U}^* = I$.

Definition 4 (*Vinnicombe, 2000*)**.** Given $P, C \in \mathcal{R}$. Let $\{N, M\}$ be a normalized rcf of P and $\{\tilde{U}, \tilde{V}\}$ be a normalized lcf of C. Then $G := \begin{bmatrix}N\\ M\end{bmatrix}$ and $\tilde{K} := \begin{bmatrix}-\tilde{U} & \tilde{V}\end{bmatrix}$ where G is referred to as the normalized right graph symbol of P, and $\tilde{K}$ is the normalized inverse left graph symbol of C and satisfy $G^*G = I$ and $\tilde{K}\tilde{K}^* = I$.

Definition 5 (*McGowan & Kuc, 1982*)**.** The unwrapped phase of a transfer function is denoted by **unwarg** and refers to the phase of the frequency response when it is in the form of a continuous function of frequency.

The experimental set-up used in Dehghani et al. (2009) is presented here which will also be used for testing robust performance improvement, the main concern and key proposition of this paper. For testing, the controller C_0 is implemented in 'observer form' as depicted in Fig. 2(a) (Vinnicombe, 2000) using a left coprime factorization of the controller $C_0 = \tilde{V_0}^{-1}\tilde{U_0}$ where the factor $\tilde{V_0}^{-1}$ is implemented in forward path and $\tilde{U_0}$ is placed in feedback path of the closed-loop system.

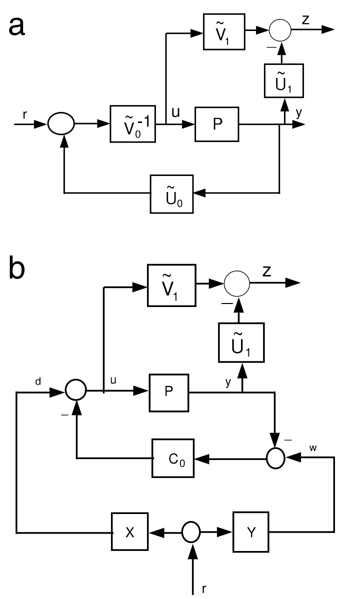

Fig. 2. Experimental set-up.

If a controller is not already implemented in this observer form, but simply implemented as in Fig. 1, then one could use the injection of exogenous signals ω and d before and after the controller, as shown in Fig. 2(b), to produce an equivalent observer form implementation. Note that $\begin{pmatrix} X \\ Y \end{pmatrix}$ are any stable transfer function (or filters) that satisfy the Bézout identity with a left coprime factorization of C_0 (i.e. $[-\tilde{U}_0 \; \tilde{V}_0]\begin{bmatrix} X \\ Y \end{bmatrix} = I$)—see Dehghani et al. (2009) for details.

The plant transfer function P is assumed to be unknown but available for data collection onto the physical closed loop. The following theorem shows the necessary and sufficient conditions for testing internal stability of a new alternative controller C_1.

Theorem 6 (*Dehghani et al., 2009; Lanzon et al., 2006*). *Given controllers $C_0, C_1 \in \mathcal{R}$ and assume $[P, C_0]$ is internally stable on a physical plant $P \in \mathcal{R}$. Let $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$ be left coprime factorizations over $\mathcal{RH}_\infty$. Let T be the stable mapping $r \mapsto z$ in Fig. 2(a) or Fig. 2(b). Then the following statements are equivalent: (1) $[P, C_1]$ is internally stable; (2) $T^{-1} \in \mathcal{RH}_\infty$; (3) $\mathbf{det}\, T(j\omega) \neq 0 \;\forall\omega$ and $\mathbf{wno}\, \mathbf{det}\, T = 0$; (4) $\mathbf{det}\, T(j\omega) \neq 0 \;\forall\omega$ and $\mathbf{unwrag}\, \mathbf{det}\, T(j\infty) = \mathbf{unwarg}\, \mathbf{det}\, T(j0)$ where $\mathbf{unwarg}(.)$ denotes the unwrapped phase of a scalar transfer function as in Definition 5.*

For the closed-loop data-based stability tests, the following two assumptions were made:

Assumption 1. The factors $\tilde{V}_0$ and $\tilde{V}_1$ are chosen such that $\tilde{V}_0(j\infty) = \tilde{V}_1(j\infty) = I$.

Assumption 2. The transfer functions PC_0 and PC_1 are strictly proper.

Assumption 1 is without loss of generality and Assumption 2 is very mild and can be easily satisfied in practice. The following formal relation (see Lemma 11 in Dehghani et al., 2009) underpins the main results of this paper.

Since

$$T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1} \tag{1}$$

then

$$T' = T - I = \left[-(\tilde{U}_1 - \tilde{U}_0) \quad (\tilde{V}_1 - \tilde{V}_0)\right] \times \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} = (\tilde{K}_1 - \tilde{K}_0)G(\tilde{K}_0 G)^{-1}. \tag{2}$$

From (1), we can rewrite $T = \tilde{V}_1(I - C_1P)(I - C_0P)^{-1}\tilde{V}_0^{-1}$. Then by Assumptions 1 and 2, it is evident that at high frequency T tends to I, i.e. from (2), T' is strictly proper. This trick simplifies the experiment significantly and indicates that experiments need not be performed on the whole frequency range to characterize the closed-loop system T, but only up to some finite frequency (i.e. bandwidth) ω_0; see Dehghani et al. (2009) for detail.

3. Testing stabilizing controllers for robust performance improvement

In the previous section, an experimental set-up was described (see Fig. 2(a) and (b)) for testing robust stability conditions. Once it is known that the stability conditions are satisfied, an immediately subsequent important question is raised: Does this stabilizing controller improve robust performance of the closed-loop system or not? In this section, new experiments are proposed to answer this last question. We will use the same experimental set-up as shown in Fig. 2(a) and (b) to test for robust performance improvement.

We now define the robust stability margin (Lanzon & Papageorgiou, 2009; Vinnicombe, 2000) for the interconnected systems shown in Fig. 1 as follows:

$$b(P, C) = \begin{cases} \left\| \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix} \right\|_\infty^{-1} \\ \quad \text{when } [P, C] \text{ is internally stable,} \\ 0 \quad \text{otherwise.} \end{cases} \tag{3}$$

Using normalized graph symbols, we can write
$b(P, C) = \|G(\tilde{K}G)^{-1}\tilde{K}\|_\infty^{-1} = \|(\tilde{K}G)^{-1}\|_\infty^{-1}$.
Hence the generalized robust stability margin $b(P, C)$ can now be equivalently represented as follows:

$$b(P, C) = \left[\sup_\omega \bar{\sigma}\left(\left(\tilde{K}(j\omega)G(j\omega)\right)^{-1}\right)\right]^{-1} = \inf_\omega \underline{\sigma}\left(\tilde{K}(j\omega)G(j\omega)\right)$$

when $[P, C]$ is internally stable. Also define $\rho(P(j\omega), C(j\omega)) = \underline{\sigma}(\tilde{K}(j\omega)G(j\omega))$ to be the pointwise in frequency generalized robust stability margin. Note that, pointwise in frequency robust performance $\rho(P(j\omega), C(j\omega))$ is always greater or equal to $b(P, C)$, robust performance over all frequencies.

The generalized robust stability margin $b(P, C)$ is a measure of robust performance, not just robust stability, of the closed-loop system (Lanzon & Papageorgiou, 2009; McFarlane & Glover, 1992; Vinnicombe, 2000; Zhou et al., 1996). This is because the closed-loop transfer function can be bounded in terms of this number and weighting functions (McFarlane & Glover, 1992; Zhou et al., 1996). A higher value of $b(P, C)$ indicates a higher level of robust performance. This means that when an existing controller is replaced by a new attractive stabilizing controller in the closed-loop system, an increase in $b(P, C)$ implies an improvement in robust performance. In this section, new test methods will be proposed to check whether $b(P, C)$ increases or not so that we can ascertain whether robust performance improves or not for the closed-loop system. Throughout this paper, we denote the existing controller and the new attractive stabilizing controller which we would like to test by, C_0 and C_1 respectively. The performance improvement conditions are given here both pointwise in frequency and over all frequencies.

3.1. Performance improvement pointwise in frequency

Consider the experimental set-up shown in Fig. 2(a) or Fig. 2(b). The transfer function matrices $T : r \mapsto z$ and $T' : r \mapsto (z - r)$ are given in (1) and (2), respectively, though neither of these two transfer functions can be computed explicitly during the experiment as P is not known to the designer. Here, $\tilde{K}_1$ and $\tilde{K}_0$ are respectively the normalized inverse left graph symbols of C_1 and C_0 and G is the normalized right graph symbol of P (see Definition 4). From the above relation, we have $(\tilde{K}_1 G) = T(\tilde{K}_0 G)$. Using singular value inequalities Green and Limebeer (1995), the above expression can be rewritten pointwise in frequency as

$$\underline{\sigma}(T(j\omega))\underline{\sigma}(\tilde{K}_0 G(j\omega)) \leq \underline{\sigma}(\tilde{K}_1 G(j\omega)) \leq \bar{\sigma}(T(j\omega))\underline{\sigma}(\tilde{K}_0 G(j\omega)). \quad (4)$$

3.1.1. Pointwise in frequency sufficient condition

If $\underline{\sigma}(T(j\omega)) > 1\ \forall\omega$, then $\underline{\sigma}(\tilde{K}_1 G(j\omega)) > \underline{\sigma}(\tilde{K}_0 G(j\omega))\ \forall\omega$, which means we have pointwise improvement in $\rho(P(j\omega), C(j\omega))$. Checking this sufficient condition pointwise in frequency is equivalent to checking the condition: $\tilde{z}^*(j\omega)\tilde{z}(j\omega) > \tilde{r}^*(j\omega)\tilde{r}(j\omega)\ \forall\omega, \forall\tilde{r}(j\omega) \in \mathbb{C}^m, \tilde{r}(j\omega) \neq 0$ as $\underline{\sigma}(A) = \inf_{x\neq 0} \frac{\|Ax\|_2}{\|x\|_2}$ and $\|x\|_2^2 = x^* x$.

3.1.2. Pointwise in frequency necessary condition

In order to have any hope for the desired pointwise in frequency robust performance improvement to be achieved via an increase in $\rho(P(j\omega), C(j\omega))$, we necessarily need $\bar{\sigma}(T(j\omega)) > 1\ \forall\omega$. Checking this condition is equivalent to checking the condition: $\forall\omega, \exists\tilde{r}(j\omega) \in \mathbb{C}^m, \tilde{r}(j\omega) \neq 0 : \tilde{z}^*(j\omega)\tilde{z}(j\omega) > \tilde{r}^*(j\omega)\tilde{r}(j\omega)$ as $\bar{\sigma}(A) = \sup_{x\neq 0} \frac{\|Ax\|_2}{\|x\|_2}$ and $\|x\|_2^2 = x^* x$.

Remark 1. The pointwise in frequency necessary condition is easier to test than the pointwise in frequency sufficient condition because for the necessary condition, one needs to find only one pointwise signal $\tilde{r} \in \mathbb{C}^m, \tilde{r} \neq 0$ that results in amplification of signal norms, whereas for the sufficient condition one needs to check that all signals $\tilde{r} \in \mathbb{C}^m, \tilde{r} \neq 0$ result in amplification of signal norms.

3.2. Performance improvement over all frequencies

Inequality (4) implies (by taking the appropriate infimum and supremum in the correct orders)

$$\left[\inf_{\omega} \underline{\sigma}(T(j\omega))\right] b(P, C_0) \leq b(P, C_1) \leq \left[\sup_{\omega} \bar{\sigma}(T(j\omega))\right] b(P, C_0)$$
$$\Rightarrow \frac{b(P, C_0)}{\|T^{-1}\|_\infty} \leq b(P, C_1) \leq \|T\|_\infty b(P, C_0). \quad (5)$$

Since $[P, C_0]$ and $[P, C_1]$ are both internally stable, from Theorem 6 we have $T, T^{-1} \in \mathcal{RH}_\infty$ where T and T' are defined in (1) and (2). Then, $T^{-1} : z \mapsto r$ and $\|T\|_\infty = \sup_{r\in\mathcal{L}_2[0,\infty), r\neq 0} \frac{\|z\|_{\mathcal{L}_2}}{\|r\|_{\mathcal{L}_2}}$ and $\|T^{-1}\|_\infty = \sup_{z\in\mathcal{L}_2[0,\infty), z\neq 0} \frac{\|r\|_{\mathcal{L}_2}}{\|z\|_{\mathcal{L}_2}} = \sup_{r\in\mathcal{L}_2[0,\infty), r\neq 0} \frac{\|r\|_{\mathcal{L}_2}}{\|z\|_{\mathcal{L}_2}}$ since T is a unit in $\mathcal{RH}_\infty$ (i.e. bijective on $\mathcal{L}_2[0, \infty)$).

3.2.1. Non-pointwise sufficient condition

If $\|T^{-1}\|_\infty < 1$, then $b(P, C_1) > b(P, C_0)$ which is the desired improvement in robust performance. Checking this non-pointwise sufficient condition $\|T^{-1}\|_\infty < 1$ is equivalent to checking the condition: $\int_0^\infty (z(t)^* z(t) - r(t)^* r(t))\, dt > 0\ \forall r \in \mathcal{L}_2[0, \infty), r \neq 0 \Leftrightarrow \|z\|_{\mathcal{L}_2} > \|r\|_{\mathcal{L}_2} \forall r \in \mathcal{L}_2[0, \infty), r \neq 0$.

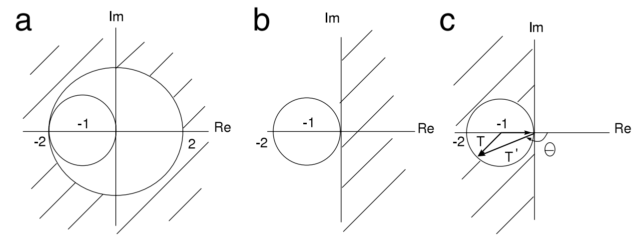

Fig. 3. Complex plane.

3.2.2. Non-pointwise necessary condition

For having any hope of achieving $b(P, C_1) > b(P, C_0)$ (i.e. achieving robust performance improvement), we need the necessary condition $\|T\|_\infty > 1$. The test of this necessary condition $\|T\|_\infty > 1$ is equivalent to testing the condition: $\exists r \in \mathcal{L}_2[0, \infty), r \neq 0 : \int_0^\infty (z(t)^* z(t) - r(t)^* r(t))\, dt > 0 \Leftrightarrow \exists r \in \mathcal{L}_2[0, \infty), r \neq 0 : \|z\|_{\mathcal{L}_2} > \|r\|_{\mathcal{L}_2}$.

Similar to Remark 1, it is worth noting that the necessary condition is easy to test because it involves finding just one (any) energy bounded input r which achieves signal amplification. This is in contrast with the sufficient condition which requires checking that signal amplification occurs for all bounded-energy inputs r.

The following necessary and sufficient result in a SISO setting allows us to check a priori robust performance improvement for a new stabilizing controller C_1 if it were to be inserted into the closed-loop system and all the tests performed without actually replacing C_0 by C_1.

Theorem 7. *Let the suppositions of Theorem 6 and Assumptions 1 and 2 hold and furthermore, $T^{-1} \in \mathcal{RH}_\infty$ and define $\tilde{z}' = \tilde{z} - \tilde{r}$. Then $|T(j\omega)| > 1 \Leftrightarrow |\tilde{z}'(j\omega)| > 2|\tilde{r}(j\omega)| \cos[\pi - (\angle\tilde{z}'(j\omega) - \angle\tilde{r}(j\omega))]$. Consequently, $\{\omega : T(j\omega) > 1\} \equiv \{\omega : |\tilde{z}'(j\omega)| > 2|\tilde{r}(j\omega)| \cos[\pi - (\angle\tilde{z}'(j\omega) - \angle\tilde{r}(j\omega))]\}$.*

Proof. Since $T : \underset{r\mapsto z}{\mathcal{L}_2[0,\infty) \to \mathcal{L}_2[0,\infty)}$ and $T = I + T'$, then $T' : \underset{r\mapsto z'}{\mathcal{L}_2[0,\infty) \to \mathcal{L}_2[0,\infty)}$ which yields $T'(j\omega) = \frac{\tilde{z}'(j\omega)}{\tilde{r}(j\omega)}$ and consequently $|T(j\omega)| > 1 \Leftrightarrow |1 + T'(j\omega)| > 1$.

Now via Fig. 3(c), it is simple trigonometry to show that $|T(j\omega)| = 1 \Leftrightarrow |1 + T'(j\omega)| = 1 \Leftrightarrow 1 = 1 + |T'(j\omega)|^2 - 2|T'(j\omega)| \cos(\pi - \theta)$ for $\theta = \angle T'(j\omega) = \angle\tilde{z}'(j\omega) - \angle\tilde{r}(j\omega) \in (\frac{\pi}{2}, \pi)$ OR $(-\pi, -\frac{\pi}{2})$ (via cosine rule for triangle) $\Leftrightarrow |\tilde{z}'(j\omega)| = 2|\tilde{r}(j\omega)| \cos[\pi - (\angle\tilde{z}'(j\omega) - \angle\tilde{r}(j\omega))]$. The result then follows. □

Corollary 1. *Given the suppositions of Theorem 7. Then the following two statements hold:* (a) $|T(j\omega)| > 1$ *if* $|\angle\tilde{z}'(j\omega) - \angle\tilde{r}(j\omega)| \leq \frac{\pi}{2}$; (b) $|T(j\omega)| > 1$ *if* $|\tilde{z}'(j\omega)| > 2|\tilde{r}(j\omega)|$.

Proof. Trivial via Theorem 7 statement on Fig. 3(a) and (b). □

This theorem gives the necessary and sufficient condition for improvement in $b(P, C)$ that in turn indicates the improvement of robust performance as well as the robust stability margin. Note that the above theorem is only applicable to SISO linear time-invariant systems. In the following theorem, sufficient conditions for robust performance improvement are given for MIMO linear time-invariant systems.

Theorem 8. *Let the suppositions of Theorem 6 and Assumptions 1 and 2 hold and furthermore, $T^{-1} \in \mathcal{RH}_\infty$ and define $T' = T - I$. Then $\underline{\sigma}(T(j\omega)) > 1$ if $\underline{\sigma}(T'(j\omega)) > 2$ or $T'(j\omega) + T'(j\omega)^* > 0$.*

Proof. Two sufficient conditions are proved separately as follows:

(a) Note that $\underline{\sigma}(T) = \underline{\sigma}(I + T') \geq \underline{\sigma}(T') - 1$. If $\underline{\sigma}(T'(j\omega)) > 2$, then $\underline{\sigma}(T(j\omega)) > 1$.

(b) Note that $T^*T = (I+T')^*(I+T') = I + T' + T'^* + T'^*T'$. Since $T'(j\omega)^*T'(j\omega) \geq 0$, if $T'(j\omega) + T'(j\omega)^* > 0$, then $T^*T > I$, i.e., $\underline{\sigma}(T(j\omega)) > 1$. □

In the above theorem, two sufficient conditions are presented for pointwise in frequency improvement of $b(P, C)$ for MIMO linear time-invariant systems. However, the first condition is impossible to satisfy when the controller change is small whereas the second condition can still be rather easily be fulfilled. The second condition also has a profound philosophical implication—that as long as the controller change is in the correct direction, then the controller change does not need to be small. Indeed, it can be arbitrarily large in the correct direction of fulfillment of $T'(j\omega) + T'(j\omega)^* > 0$ and robust performance improvement is still guaranteed. Checking the second condition is equivalent to check a necessary and sufficient condition which is presented in the following theorem.

Theorem 9. *Let the suppositions of Theorem 6 and Assumptions 1 and 2 hold and furthermore, $T^{-1} \in \mathcal{RH}_\infty$. Given $T = (\tilde{K}_1G)(\tilde{K}_0G)^{-1}$ and $T' = T - I$. Then $T'(j\omega) + T'(j\omega)^* > (\geq) 0$ if and only if*

$$(\tilde{K}_1G)^*(\tilde{K}_1G)(j\omega) - (\tilde{K}_0G)^*(\tilde{K}_0G)(j\omega)$$
$$> (\geq)[(\tilde{K}_1 - \tilde{K}_0)G]^*[(\tilde{K}_1 - \tilde{K}_0)G](j\omega). \quad (6)$$

Proof. This theorem is proved via sequence of equivalent steps: $T'(j\omega) + T'(j\omega)^* > 0 \Leftrightarrow T(j\omega)^*T(j\omega) - I > T'(j\omega)^*T'(j\omega) \Leftrightarrow (\tilde{K}_0G)^{-*}(\tilde{K}_1G)^*(\tilde{K}_1G)(\tilde{K}_0G)^{-1} - I > (\tilde{K}_0G)^{-*}[(\tilde{K}_1 - \tilde{K}_0)G]^*[(\tilde{K}_1 - \tilde{K}_0)G](\tilde{K}_0G)^{-1}$. □

In inequality (6), the right hand side is related to the size of the controller change and the left hand side is the difference between the new and the old robust stability margins. This condition states that for the controller change to yield a change in the positive-real direction (i.e. $T'(j\omega) + T'(j\omega)^* > 0$) which then guarantees robust performance improvement, we need the controller change to be such that it has larger impact on the increase in $b(P, C)$ than it has on the size of the transfer function $T'(j\omega)$. This is needed so that the left hand side is greater than the right hand side in inequality (6).

In safe adaptive control, if one is close to the critical Nyquist point and also has no information on which direction to perform a controller change, it is always better to make small changes on the controller. These kinds of results then can only give a lower bound on the maximum performance degradation and we often are content with this as an acceptable compromise to safe adaptive control algorithms and use this kind of argument to justify why one should make small steps so that we do not inadvertently lose stability. But if one is not completely lacking all information and can perform the tests in this paper, then there is a large set of directions where huge controller changes are perfectly acceptable and indeed yield performance and stability margin improvement. This means we are allowed to take arbitrary huge steps that satisfy condition (6), which corresponds to a step in the positive-real direction and still attain robust performance improvement. Note that this is equivalent to $\langle z', r\rangle_{\mathcal{L}_2} \geq 0\ \forall r \in \mathcal{L}_2$ where $z' = z - r$ and $z = Tr$.

4. Closed-loop data-based tests for performance improvement

In Theorem 7, necessary and sufficient conditions are given for SISO linear time-invariant systems to improve robust performance when an existing controller is replaced by a new attractive stabilizing controller in the closed loop. For the same objective, in Theorem 8 a sufficient condition is given for MIMO linear time-invariant systems. It is however practically unrealistic to perform experiments for all frequencies as well as for all signals in $\mathcal{L}_2$ space to check such conditions. To circumvent this difficulty, experimental procedures are proposed in this section based on the closed-loop set-up shown in Fig. 2(a) or Fig. 2(b).

In Dehghani et al. (2009) (see Theorems 10 and 12), two novel experiments have been proposed based on closed-loop measured data to validate controllers for internal stability. The first experiment proposed in Dehghani et al. (2009) was a falsification test for internal stability and interestingly, the same falsification test data collected during the experiment can be reprocessed to also check for robust performance improvement of the closed-loop system. This experiment significantly reduces the experimental effort as well as utilizing an extremely simple test procedure.

We construct $\bar{Z}$ by following Dehghani et al. (2009) based on the experimental set-up shown in Fig. 2(a) or Fig. 2(b). If $\mathbf{det}\ \bar{Z} \leq 0$, then the controller does not internally stabilize the closed loop. Assuming that internal stability of the new attractive controller has already been established via the tools and experiments in Dehghani et al. (2009) and Lanzon et al. (2006), we can then use $\bar{Z}$ to check whether this stabilizing controller will improve $b(P, C)$ of the closed-loop system. Note that, for both SISO and MIMO linear time-invariant systems, a sufficient condition for improving $b(P, C)$ is the positive-realness of T' where $T' = T - I$. In this regard, the following test for positive-realness of T' at low frequency is useful.

Theorem 10. *Let the suppositions of Theorem 6 and Assumptions 1 and 2 hold and furthermore, $T^{-1} \in \mathcal{RH}_\infty$. Let e_i denote a reference signal where a step is applied at the ith input while the other inputs are kept as 0. Perform n experiments with reference signal $r(t) = e_i(t),\ i = 1, \ldots, n$ and let $\bar{z}_i$ be the steady state output of the map $T : r \mapsto z$ recorded in each experiment. Define $\bar{Z} = [\bar{z}_1, \ldots, \bar{z}_n]$. Then $\exists \omega_1 > 0 : T'(j\omega) + T'(j\omega)^* > 0\ \forall \omega \in [0, \omega_1] \Leftrightarrow \bar{Z} + \bar{Z}^T > 2I$.*

Proof. We have $T = I + T'$ and $\bar{Z} = T(0)$. Hence, $T'(0) + T'(0)^* = T(0) + T(0)^* - 2I = \bar{Z} + \bar{Z}^T - 2I$. The result then follows by noting that $T'(j\omega)$ is a continuous function of ω. □

If $\bar{Z} + \bar{Z}^T \not> 2I$, then $T'(j\omega)$ cannot be positive-real for all frequencies. Since the positive-realness of T' is a sufficient condition for robust performance improvement, one cannot imply anything about the improvement of $b(P, C)$ or not when the above test condition fails. However, fulfillment of the above test condition guarantees the improvement of robust performance at zero frequency and its neighborhood.

To check the conditions of Theorems 7 and 8, an experiment will be performed up to the frequency ω_0 such that $|T'|$ is much smaller than unity for all $\omega > \omega_0$. Since $T = I + T'$, the collected closed-loop data up to the frequency ω_0 will be sufficient to characterize the required properties onto the system T as $|T| \approx 1$ when $|T'| \ll 1$.

It is indeed correct that positive-realness of T' is a sufficient condition for performance improvement. If one contends that an appropriate, though possibly conservative, test for performance improvement is positive-realness of T', then one seeks to find ways of checking whether positive-realness of T' holds or not.

Positive-realness is a condition that needs to be checked on the full frequency axis. The approach we take in this paper is to split the full frequency axis in subparts and apply 'easier' to compute conditions on each subpart. Any test on a sub-frequency region will not be sufficient to infer positive-realness of T', but when all tests on all sub-frequency regions are jelled together, the tests can cover the full frequency axis. It is also not the case that the boundaries of the tests for each sub-frequency region be known exactly. Quite to the contrary, the only requirement is that the tests together cover the full frequency axis and hence there can be considerable conservativeness in the estimation of the frequency boundaries for each sub-frequency region test.

5. Simulation example

Although the proposed experiments are based on the unknown plant, for simulation purposes, the plant transfer function is considered as known. We consider the SISO example which was presented in Dehghani et al. (2009) to validate controllers for internal stability. For the same example, test methods proposed in this paper will be performed to check whether the new controller also improves performance when it is to be inserted into the closed-loop system. The tests performed use the existing feedback interconnection with the old controller and determine whether performance improvement will happen or not before it is actually inserted in the feedback loop.

Let a SISO plant which is not known to the designer be given by $P = \frac{-186.66(s-5)(s+4.5)}{(s+10)^2(s+7)(s+6)}$. A stabilizing controller

$$C_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.6s+19.84)(s^2-0.603s+5.34)}$$

is physically connected to the plant in closed loop. In Dehghani et al. (2009), it is shown that an attractive new controller

$$C_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+2)(s^2+2.26s+3.52)}$$

also ensures internal stability of the closed-loop system if C_0 were to be replaced by C_1 and this conclusion is drawn on the basis of data collected via experiments performed on the closed loop of the unknown plant P and the original controller C_0, without inserting C_1 in the feedback loop, using C_1 to only filter the collected data instead (see Dehghani et al., 2009 and Lanzon et al., 2006 for details). Left coprime factors over $\mathcal{RH}_\infty$ of the known controllers are, respectively, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$ where

$$\tilde{V}_0 = \frac{(s^2+8.603s+19.84)(s^2-0.602s+5.34)}{(s^2+8.64s+19.97)(s^2+1.83s+6.96)},$$

$$\tilde{U}_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.64s+19.97)(s^2+1.83s+6.96)},$$

$$\tilde{V}_1 = \frac{(s+2)(s^2+2.26s+3.52)}{(s+1.87)(s^2+2.81s+3.712)},$$

$$\tilde{U}_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+1.87)(s^2+2.81s+3.712)}.$$

$\tilde{V}_0$ and $\tilde{V}_1$ satisfy the Assumption 1. Assumption 2 is automatically satisfied due to strictly proper P and proper controllers. To check performance improvement, we do the following experiments.

(a) *Falsification test*: For the experimental set-up shown in Fig. 2(a) or Fig. 2(b), we record steady state response at the output when the injected input is a unit step and it is observed that $\bar{Z} = 5.68$. Since $\bar{Z} + \bar{Z}^T > 2I$, this test does not falsify the necessary and sufficient condition for strictly positive-realness of T' at DC and its neighborhood frequency. Consequently, no conclusion can be drawn and more experimental effort is required in this case to determine whether this new controller improves robust performance or not.

(b) In the experimental set-up shown in Fig. 2(a) or Fig. 2(b), we do a sine-sweep starting from DC frequency and the corresponding magnitude plot of T' is shown in Fig. 4. Notice that the exact plot is irrelevant as we need only some key properties and points on this curve to characterize the transfer function T. Beyond 30 rad/s, $|T'| \approx 0$ and so $|T| \approx 1$ as $T = I + T'$. This means we only need to test up to approximately 30 rad/s.

Also, if the sine-sweep confirms confidently that $|T'| > 2$ up to approximately 0.9 rad/s, as indeed depicted in Fig. 4, then we know via Corollary 1 that up to 0.9 rad/s we have robust performance improvement.

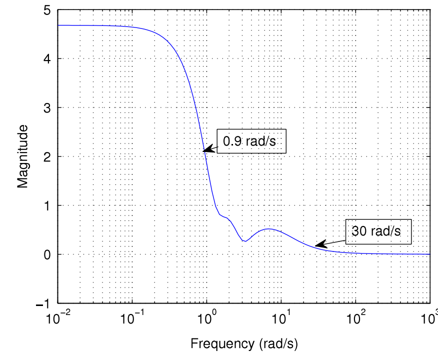

Fig. 4. Magnitude plot of T'.

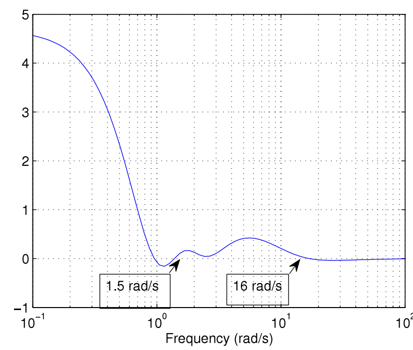

Fig. 5. $\mathrm{Re}[\tilde{z}'(j\omega)^*\tilde{r}(j\omega)]$ vs. frequency.

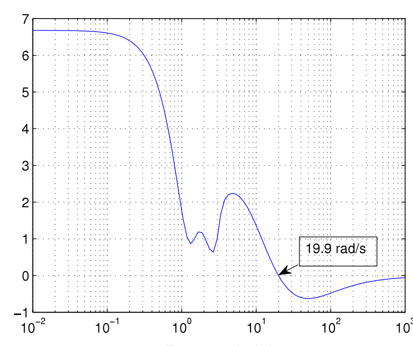

Fig. 6. $|T'(j\omega)| - 2\cos(\pi - \angle T'(j\omega))$ vs. frequency.

(c) From the sine-sweep data beyond 0.9 rad/s, compute also $\mathrm{Re}[\tilde{z}'(j\omega)^*\tilde{r}(j\omega)]$. This is plotted in Fig. 5. The exact frequency plot is irrelevant as the information that needs to be extracted only uses a few key data points.

From this experiment, we can confidently conclude that there is robust performance improvement also in the frequency range (1.5, 16 rad/s) as $\mathrm{Re}[\tilde{z}'(j\omega)^*\tilde{r}(j\omega)] \geq 0$ in this frequency range implying $T'(j\omega) + T'(j\omega)^* \geq 0$.

(d) Consequently, robust performance improvement is guaranteed in [0, 0.9 rad/s) and (1.5, 16 rad/s) via the preceding tests. The question of whether robust performance improvement happens also in the frequency intervals [0.9, 1.5 rad/s] and [16, 30 rad/s] cannot be answered without the precise data having experimental effort related to Theorem 7 as in Fig. 6. From the precise data, it is evident that the robust performance will be improved in the frequency range [0, 19.9 rad/s), while the proposed test methods give the ranges [0, 0.9 rad/s) and (1.5, 16 rad/s), and hence the methods are not very conservative.

6. Conclusions

In this paper, conditions are derived for robust performance improvement based on the closed-loop data when an existing controller is replaced by an attractive new stabilizing controller in the feedback loop. The results are applicable to both SISO and MIMO linear time-invariant systems. For the proposed tests, the plant model is assumed to be unknown. Such an assumption is common in iterative identification and control redesign techniques. A sufficient condition is derived for robust performance improvement that shows that as long as the controller change is done in the positive-real direction, such a controller change can be of an arbitrarily large size. This is contrary to common wisdom adopted in the mainstream adaptive control literature where small controller changes are adopted in order for such a controller change to be cautious or safe. The experimental set-up used in this paper is identical to (Dehghani et al., 2009; Lanzon et al., 2006); however, the proposed conditions for robust performance improvement cannot be obtained from the test results of Dehghani et al. (2009) and Lanzon et al. (2006) directly. The numerical example in this paper demonstrates the applicability of the proposed test conditions.

References

Anderson, B. D. O. (2004). Two decades of adaptive control pitfalls. In *Proceedings of the 8th international conference on control, automation, robotics and vision.* Kunming, Chaina, December.

Anderson, B. D. O., & Gevers, M. (1998). Fundamental problems in adaptive control. In D. Normand-Cyrot (Ed.), *Perspectives in control theory and application*. Berlin: Springer-Verlag.

Baldi, S., Battistelli, G., Mosca, E., & Tesi, P. (2010). Multi-model unfalsified adaptive switching supervisory control. *Automatica*, (2), 249–259.

Bitmead, R. R. (1993). Iterative control design approaches. In *Proceedings of the 12th IFAC world congress.* Sydney, Australia (pp. 381–384).

Date, P., & Lanzon, A. (2004). A combined iterative scheme for identification and control redesigns. *International Journal of Adaptive Control and Signal Processing*, (8), 629–644.

De Callafon, R. A., & Van Den Hof, P. M. J. (1997). Suboptimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of Systems*, (1), 77–101.

Dehghani, A., Lanzon, A., & Anderson, B. D. O. (2004). An H_∞ algorithm for the windsurfer approach to adaptive robust control. *International Journal of Adaptive Control and Signal Processing*, (8), 607–628.

Dehghani, A., Lecchini, A., Lanzon, A., & Anderson, B. D. O. (2009). Validating controllers for internal stability utilizing closed-loop data. *IEEE Transactions on Automatic Control*, (11), 2719–2725.

Gevers, M. (2000). A decade of progress in iterative control design: from theory to practice. In *Symposium on advanced control of chemical processes.* Pisa, Italy (pp. 677–694).

Gevers, M. (2002). Modelling, identification and control. In P. Albertos, & A. Sala (Eds.), *Iterative identification and control*. London: Springer-Verlag.

Gevers, M., Bombois, X., Codrons, B., Scorletti, G., & Anderson, B. D. O. (2003). Model validation for control and controller validation in a prediction error identification framework—part I: theory. *Automatica*, (3), 403–415.

Green, M., & Limebeer, D. J. N. (1995). *Linear robust control.* Prentice Hall.

Hildebrand, R., Lecchini, A., Solari, G., & Gevers, M. (2005). Asymptotic accuracy of iterative feedback tuning. *IEEE Transactions on Automatic Control*, (8), 1182–1185.

Hjalmarsson, H., Gevers, M., Gunnarson, S., & Lequin, O. (1998). Iterative feedback tuning: theory and applications. *IEEE Control Systems Magazine*, *18*(4), 26–41.

Kammer, L. C., Bitmead, R. R., & Bartlett, P. L. (2000). Direct iterative tuning via spectral analysis. *Automatica*, (9), 1301–1307.

Lanzon, A., Leccchini, A., Dehghani, A., & Anderson, B. D. O. (2006). Checking if controllers are stabilizing using closed-loop data. In *Proceedings of the 45th IEEE conference on decision and control*.

Lanzon, A., & Papageorgiou, G. (2009). Distance measures for uncertain linear systems: a general theory. *IEEE Transactions on Automatic Control*, (7), 1532–1547.

Lecchini, A., Lanzon, A., & Anderson, B. D. O. (2006). A model reference approach to safe controller changes in iterative identification and control. *Automatica*, (2), 193–203.

Manuelli, C., Cheong, S. G., Mosca, E., & Safonov, M. G. (2007). Stability of unfalsified adaptive control with non-SCLI controllers and related performance under different prior knowledge. In *Proceedings of the European control conference.* Kos, Greece. July (pp. 702–708).

McFarlane, D., & Glover, K. (1992). A loop shaping design procedure using H_∞ synthesis. *IEEE Transactions on Automatic Control*, (6), 759–769.

McGowan, R., & Kuc, R. (1982). A direct relation between a signal time series and its unwrapped phase. *IEEE Transactions on Acoustics, Speech, & Signal Processing*, (5), 719–726.

Schrama, R. J. P. (1992). Accurate identification for control: the necessary of an iterative scheme. *IEEE Transactions on Automatic Control*, 991–994.

Schrama, R.J.P., & Van Den Hof, P.M.J. (1992). An iterative scheme for identification and control design based on coprime factorizations. In *Proceedings of the American control conference.* Chicago, IL, USA (pp. 2842–2846).

Vinnicombe, G. (2000). *Uncertainty and feedback: H_∞ loop-shaping and ν-gap metric*. Imperial College Press.

Zhou, K., Doyle, J. C., & Glover, K. (1996). *Robust and optimal control.* Prentice Hall.

Sourav Patra received the Ph.D. degree in Electrical Engineering from the Indian Institute of Technology Kharagpur, India in 2009. He is currently appointed as an Assistant Professor at the Department of Electrical Engineering, Indian Institute of Technology Kharagpur, India, having previously worked as a Postdoctoral Research Associate at the Control Systems Centre, School of Electrical and Electronic Engineering, University of Manchester, UK. Prior to this position, he was appointed as a Reader in the Department of Avionics, Indian Institute of Space Science and Technology Trivandrum, India. His major research area is robust control, and other research interests include the actuator saturator control, time-delay systems, nonlinear systems and systems biology.

Alexander Lanzon is a Reader in Control Engineering at the School of Electrical and Electronic Engineering and the Autonomous Systems Theme Leader at the Aerospace Research Institute, both at the University of Manchester. He was born in Malta. He received the B.Eng. (Hons.) degree in Electrical and Electronic Engineering with highest-level First Class Honours from the University of Malta in 1995, and his M.Phil. and Ph.D. degrees in Control Engineering from the University of Cambridge in 1997 and 2000 respectively.

Before joining the School of Electrical and Electronic Engineering at the University of Manchester in December 2006, Alexander held academic positions at the School of Aerospace Engineering, Georgia Institute of Technology, USA, and the Australian National University, Australia. Dr. Lanzon also received earlier research training at Bauman Moscow State Technical University, Russia and industrial training at ST-Microelectronics Ltd., Malta, and Yaskawa Denki Tokyo Ltd., Japan. In Australia, Alexander held also the position of senior research scientist at National ICT Australia Ltd.

Dr. Lanzon is a Subject Editor of the International Journal of Robust and Nonlinear Control, a Fellow of the IET, and a Senior Member of the IEEE and the AIAA. His current research interests include the fundamental theory of robust feedback interconnections and aerospace control applications, particularly autonomous systems (to include unmanned aerial, ground and underwater vehicles).