Optimal Speed Control of Induction Motor Based on Linear Quadratic Regulator Theory

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***Abstract*— In this paper, the optimal speed control, based on Linear Quadratic Regulator (LQR) theory, through stator field orientation (SFO) of an inverter fed induction motor is addressed. The voltage source inverter, behaving as an actuator, is operated using space vector pulse width modulation (SVPWM) technique. The state feedback system is augmented with integral output error to enhance reference tracking and disturbance rejection criteria. In addition, a feed forward gain is introduced to ensure unity dc gain in case of reference tracking. The LQR synthesis is carried out for the whole system considering the integral output error as an additional state. It is interesting to note that the structure of the control law used here resembles the classical proportional integral control law.**

Keywords*— *Induction motor (IM); Linear quadratic regulator (LQR); Speed control; Vector control.

I. Introduction

In recent years, the use of induction motors (IM) in industries as variable speed servo drive increases due to its cost effectiveness in installation as well as in maintenance. However, the smooth control of speed in IM is not that easy as DC motors, which impedes the use IM drives. With the advancement of vector control technique for IM drive, the speed control becomes relatively smoother. Among the different vector controlled speed regulation techniques of IM, the field oriented control (FOC) is recognized as the best. There are different FOC techniques namely as stator field orientation (SFO), rotor field orientation (RFO), air gap field orientation (AFO) etc. In this work, SFO is considered as it is insensitive to the rotor circuit and also stator resistance and current can be easily and most accurately measured and so does the estimation of stator flux[1]-[3].

In this paper, stator field oriented speed control is accomplished based on the optimal control technique, linear quadratic regulator (LQR). To obtain the field orientation, LQR theory is implemented to an IM multi input multi output (MIMO) model in synchronously rotating reference frame. For tracking of reference speed in conditions of disturbance and model uncertainty, integral error is introduced as an additional state. Then the LQR synthesis is carried out for the whole system along with this additional state. This work is synthesized and simulated in MAT LAB environment.

This paper is organized as follows. The mathematical model of the IM is described in section II, whereas a brief description of proposed control scheme is presented in section III. Section IV and V deals with the results and observational conclusion, respectively.

II. IM MIMO Model

The mathematical equations of IM in the synchronously rotating $(d-q)$ frame can be written as equation (1) [2],[10].

$$\frac{d}{dt}\begin{bmatrix}\omega_r\\ \phi_{ds}\\ \phi_{qs}\\ i_{ds}\\ i_{qs}\end{bmatrix}=\begin{bmatrix}-\frac{F}{J}\omega_r+\frac{3P}{4J}(\phi_{ds}i_{qs}-\phi_{qs}i_{ds})-\frac{1}{J}T_l\\ \omega_e\phi_{qs}-R_s i_{ds}+v_{ds}\\ -\omega_e\phi_{ds}-R_s i_{qs}+v_{qs}\\ \frac{\tau_r}{\sigma L_s}\phi_{ds}+\frac{\omega_r}{\sigma L_s}\phi_{qs}-\frac{\tau_s+\tau_r}{\sigma}i_{ds}+\omega_s i_{qs}+\frac{1}{\sigma L_s}v_{ds}\\ -\frac{\omega_r}{\sigma L_s}\phi_{ds}+\frac{\tau_r}{\sigma L_s}\phi_{qs}-\omega_s i_{ds}-\frac{\tau_s+\tau_r}{\sigma}i_{qs}+\frac{1}{\sigma L_s}v_{qs}\end{bmatrix} \quad (1)$$

Where, $v_{ds},v_{qs},\phi_{ds},\phi_{qs},i_{ds},i_{qs}$ are the $d-q$ components of stator voltage, stator flux linkage and stator current respectively and $\omega_r,\omega_e,\omega_s$ are the rotor, synchronous and slip speed respectively. The symbols R_s and L_s are the stator resistance and inductance, whereas R_r and L_r are that of the rotor circuit. L_m is magnetizing inductance. The stator and rotor constants γ_s,γ_r and σ are given in equation (2).

$$\gamma_s=\frac{R_s}{L_s},\gamma_r=\frac{R_r}{L_r},\sigma=1-\frac{L_m^2}{L_sL_r} \quad (2)$$

Taking $x(t)$ as the state vector, $u(t)$ as the input, $y(t)$ as the output vector and $d(t)$ as the external disturbance to the

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system, the state space nonlinear MIMO model of the IM is constructed as follows.

$$\begin{aligned} \dot{x}(t) &= f(x(t),u(t),d(t),t) \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

which is linearized about the nominal operating point (x_n, u_n, d_n) to obtain the linear model as given below.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

Where,

$$\begin{aligned} x(t) &= [\omega_r \phi_{ds} \phi_{qs} i_{ds} i_{qs}]^t \\ u(t) &= [\omega_e v_{ds} v_{qs}] \\ y(t) &= [\omega_r \phi_{ds} \phi_{qs}] \\ d(t) &= T_l \end{aligned}$$

Where, A, B, C, E are the linear system matrix, control matrix, output matrix and disturbance matrix, respectively and are obtained following small signal approximation technique as follows.

$$A = \left.\frac{\partial f(.)}{\partial x}\right|_{(x_n,u_n,d_n)} = \begin{bmatrix} -\frac{F}{J} & \frac{3P}{4J} i_{qsn} & -\frac{3P}{4J} i_{dsn} & -\frac{3P}{4J}\phi_{qsn} & \frac{3P}{4J}\phi_{dsn} \\ 0 & 0 & \omega_{en} & -R_s & 0 \\ 0 & -\omega_{en} & 0 & 0 & -R_s \\ \frac{1}{\sigma L_s}\phi_{qsn} & \frac{\tau_r}{\sigma L_s} & \frac{\omega_{rn}}{\sigma L_s} & -\frac{\tau_s+\tau_r}{\sigma} & \omega_{sn} \\ -\frac{1}{\sigma L_s}\phi_{dsn} & -\frac{\omega_{rn}}{\sigma L_s} & \frac{\tau_r}{\sigma L_s} & -\omega_{sn} & -\frac{\tau_s+\tau_r}{\sigma} \end{bmatrix},$$

$$B = \left.\frac{\partial f(.)}{\partial u}\right|_{(x_n,u_n,d_n)} = \begin{bmatrix} 0 & 0 & 0 \\ \phi_{qsn} & 1 & 0 \\ -\phi_{dsn} & 0 & 1 \\ 0 & \frac{1}{\sigma L_s} & 0 \\ 0 & 0 & \frac{1}{\sigma L_s} \end{bmatrix},$$

$$E = \left.\frac{\partial f(.)}{\partial d}\right|_{(x_n,u_n,d_n)} = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The nonlinear IM model in (1) is linearized around nominal operating point which are chosen to be

$\omega_{rn} = 150 r/s$, $\phi_{dsn} = 2.4Wb$, $\phi_{qsn} = 0$, $i_{dsn} = 6A$, $i_{qsn} = 0$, $\omega_{en} = 157 r/s$

With the above assumptions the system matrix A, input matrix B and the disturbance matrix E takes the form

$$A = \begin{bmatrix} -0.23 & 0 & -551.74 & 0 & 94.74 \\ 0 & 0 & 157 & -6.30 & 0 \\ 0 & -157 & 0 & 0 & -6.30 \\ 0 & 238.35 & 4766.95 & -314.62 & 7 \\ -38.14 & -4766.95 & 238.35 & -7 & -314.62 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1.20 & 0 & 1 \\ 0 & 31.78 & 0 \\ 0 & 0 & 31.78 \end{bmatrix},$$

$$E = \begin{bmatrix} -26.32 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

III. CONTROLLER SYNTHESIS

The state feedback control can be achieved either by using various pole placement techniques or by using optimal quadratic error minimization techniques. In this paper, the later mentioned technique is implemented in the context of linear quadratic regulator theory. The advantage of using quadratic error minimization technique is that, the control input is proportional to the squared error, thus if the error is high, minimization is faster or vice versa.

To implement LQR the performance index/criteria (PI) [9] is taken as

$$J_{LQR} = \int (x(t)^t Q x(t) + u(t)^t R u(t)) dt \tag{5}$$

Where, Q≥0 and R>0 are the weight matrices for state $x(t)$ and control input $u(t)$ respectively.

These two are user defined variables and the dynamic performance of a system can be adjusted depending on these two weight matrices. The control input for the system (4) is considered to be.

$$u(t) = -Gx(t) + Vr(t) \tag{6}$$

Where G is the steady state optimal feedback gain matrix is given by

$$G = R^{-1} B^t P \tag{7}$$

Where P is the steady state solution of the algebraic Riccati equation

$$-\dot{P}(t) = Q + A^t P(t) + P(t)A - P(t)BR^{-1}B^t P(t) \qquad (8)$$

A feed forward gain, V, is introduced to make the steady state error zero [4] by means of making command reference to output dc gain unity. V can be derived from (4) and (6) as follows:

From (4) and (6) , $\dot{x}(t)$ can be written as

$$\dot{x}(t) = (A - BG)x(t) + BVr(t) \qquad (9)$$

And thus the transfer matrix from $r(t)$ to $y(t)$ can be written as

$$\frac{Y(s)}{R(s)} = C(sI - A + BG)^{-1}BV \qquad (10)$$

So, to make the DC gain unity i.e, $\lim_{s\to 0}\frac{Y(s)}{R(s)} = 1$

$$V = -(C(A - BG)^{-1}B)^{-1} \qquad (11)$$

For disturbance rejection and to tackle with model uncertainty an integral path for output error is also provided,

$$\varepsilon(t) = \int (r(t) - y(t))dt \qquad (12)$$

It is introduced as additional state and is augmented with the state space IM model. The new augmented state space model becomes

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\varepsilon}(t)\end{bmatrix} = \begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \varepsilon(t)\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}u(t) + \begin{bmatrix}0\\ I\end{bmatrix}r(t) \qquad (13)$$

Now, the control law for (13) with respect to the performance index (5) can be written as

$$u(t) = -\begin{bmatrix}g_1 & g_2\end{bmatrix}\begin{bmatrix}x(t)\\ \varepsilon(t)\end{bmatrix} + Vr(t) \qquad (14)$$

From the observation of the control law, it can be stated that this control law resembles the classical proportional plus integral control law with g_1 as proportional gain and g_2 as the integral gain.

The controller structure is as shown below:

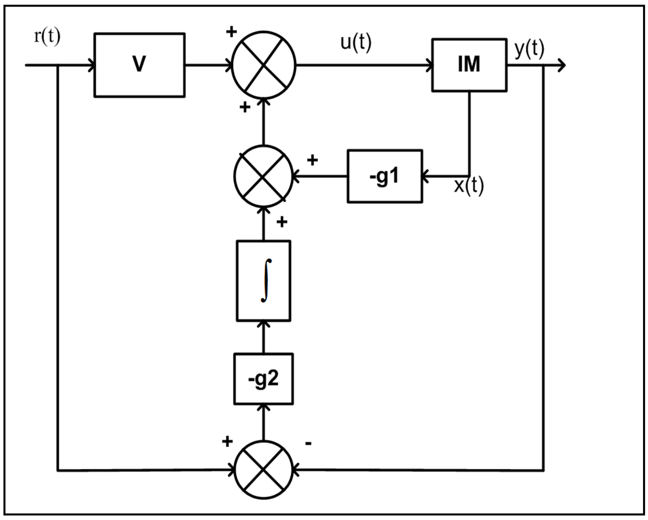

Fig 1: Block diagram of the proposed controller.

IV. SIMULATION AND RESULTS

The schematic diagram of proposed work is shown in the Fig 2. The state feedback gain synthesis and simulation work is carried out in MAT LAB environment.

The co-ordinate transformation angle θ_e , for conversion of electrical quantities from $\alpha - \beta$ to $d - q$ components is obtained from the one of the controller output, ω_e , following the mathematical expression

$$\theta_e = \int \omega_e dt \qquad (15)$$

A voltage model based estimator is used to estimate the two stator flux components in the stationery $(\alpha - \beta)$ reference frame from the equation

$$\phi_s = \int (v_s - R_s i_s)dt \qquad (16)$$

Where, $\phi_s = \phi_{ds} + j\phi_{qs}$, $v_s = v_{ds} + jv_{qs}$, $i_s = i_{ds} + ji_{qs}$

A MOSFET two level voltage source inverter (VSI) is taken for actuation. To examine the effectiveness of the proposed controller two tests are carried out, one for disturbance rejection and the other is for the speed reference tracking. For both the tests reference ϕ_{ds}, ϕ_{qs} are set at 2.4 Wb and 0 Wb respectively.

The weight matrices Q and R in the performance criteria for the augmented system are chosen as

$$Q_{8x8} = \begin{pmatrix} q_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & q_8\end{pmatrix}, R_{3x3} = \begin{pmatrix} r_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & r_3\end{pmatrix}$$

Where, q_1, q_4, q_5 are set at 100, whereas to make the flux response faster corresponding weights q_2 and q_3 are set at 10000, the weight associated with the integral error q_6, q_7, q_8 are set at 1. The weights associated with the control input r_1, r_2, r_3 are set at 50.

The VSI inverts the dc link voltage which is set at 500V, based on the $\alpha - \beta$ component of the control input voltages, through a two level SVPWM pulse generator with switching frequency 10 kHz. The control scheme is shown below:

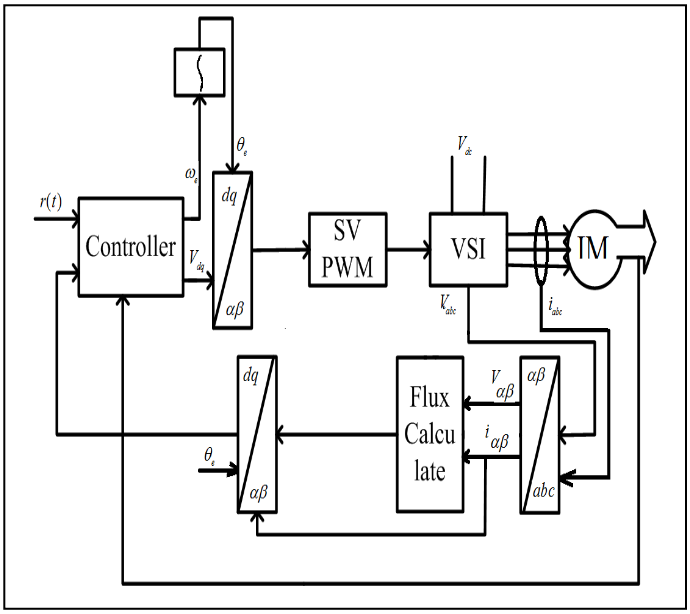

Fig 2: The proposed control scheme

TABLE 1: MOTOR SPECIFICATION

No of Pole	4
Power	1.5 kW
Voltage	380 V
Operating Frequency	50 Hz
Stator/Rotor Resistance	6.3/3.6 Ω
Stator/Rotor Inductance	0.480 H
Mutual Inductance	0.464 H
Motor Inertia	0.038 Nms^2
Friction Coefficient	0.0085 Nms

The response with no load condition and speed reference 100r/s alongwith stator flux refrences ϕ_{ds} & ϕ_{qs} at 2.4 Wb and 0 Wb, respectively is shown in Fig 3. The control scheme is shown to work satisfactorily with a settling time of 2 s.

A. Disturbance Rejection

Here mechanical load change is considered as the only source of disturbance. The load disturbance rejection is an important control objective since there will be no practical system where the load to the induction motor can be kept constant. For the test of disturbance rejection of the proposed work, the test IM is operated at 100 r/s speed reference keeping the stator fluxes ϕ_{ds} & ϕ_{qs} at their reference value 2.4 Wb and 0 Wb, respectively. The motor is started with zero load torque and then at 3 second the load torque is suddenly increased to 8 N-m and again at 6 s load torque is reduced to 3 N-m. The response is shown in Fig 4.

It is observed from Fig 3 that the proposed control scheme achieves perfect disturbance rejection with a settling time of 2 s.

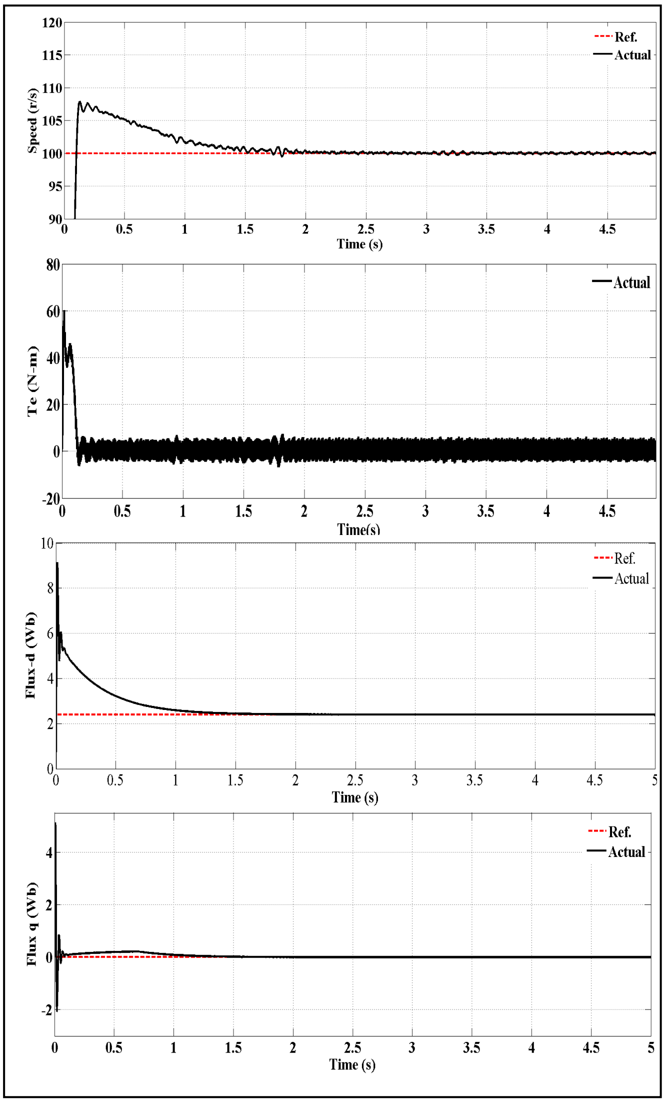

Fig 3: Nominal response with no load at 100r/s speed reference.

The speed curve has undergone an undershoot and overshoot of approximately 2% in both the cases with a sudden increase and decrease of load torque, respectively. The flux ϕ_{ds} & ϕ_{qs} are also maintained at their reference value 2.4 Wb and 0 Wb, respectively.

B. Reference Tracking

The main objective of designing a controller is to obtain a desired response of the system with taking the physical constraints into consideration. In the present case, operating the motor at different speed level and at constant $d-q$ fluxes is primary objective along with the disturbance rejection. The results are presented in the Fig 5.

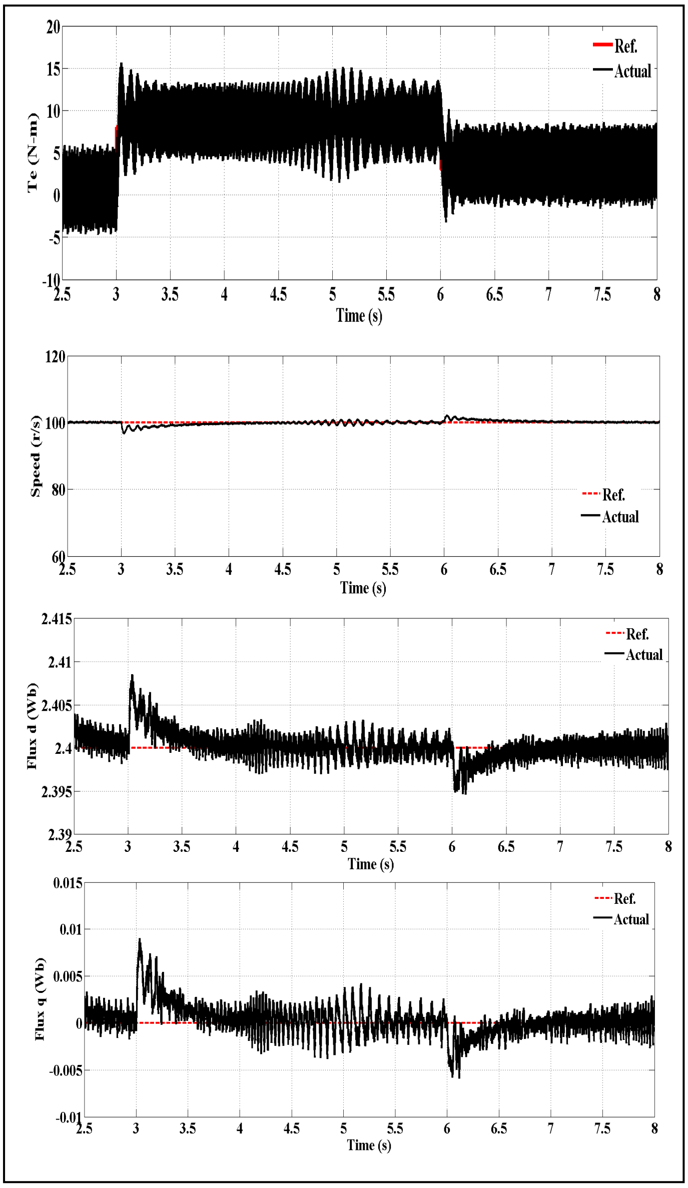

Fig 4: Enlarged electromagnetic torque, speed, and flux Vs Time with step change in mechanical load.

The motor is started with no load and the whole tracking examination is performed under no load condition.The reference speed to be tracked is set initially at 100 r/s and then at 5 s a step change in reference speed from 100 r/s to 150 r/s is introduced and again at 10 s the reference speed is brought back to 100 r/s. From the observation of the response curves, it can be verified that, the controller is able to track the reference command efficiently.

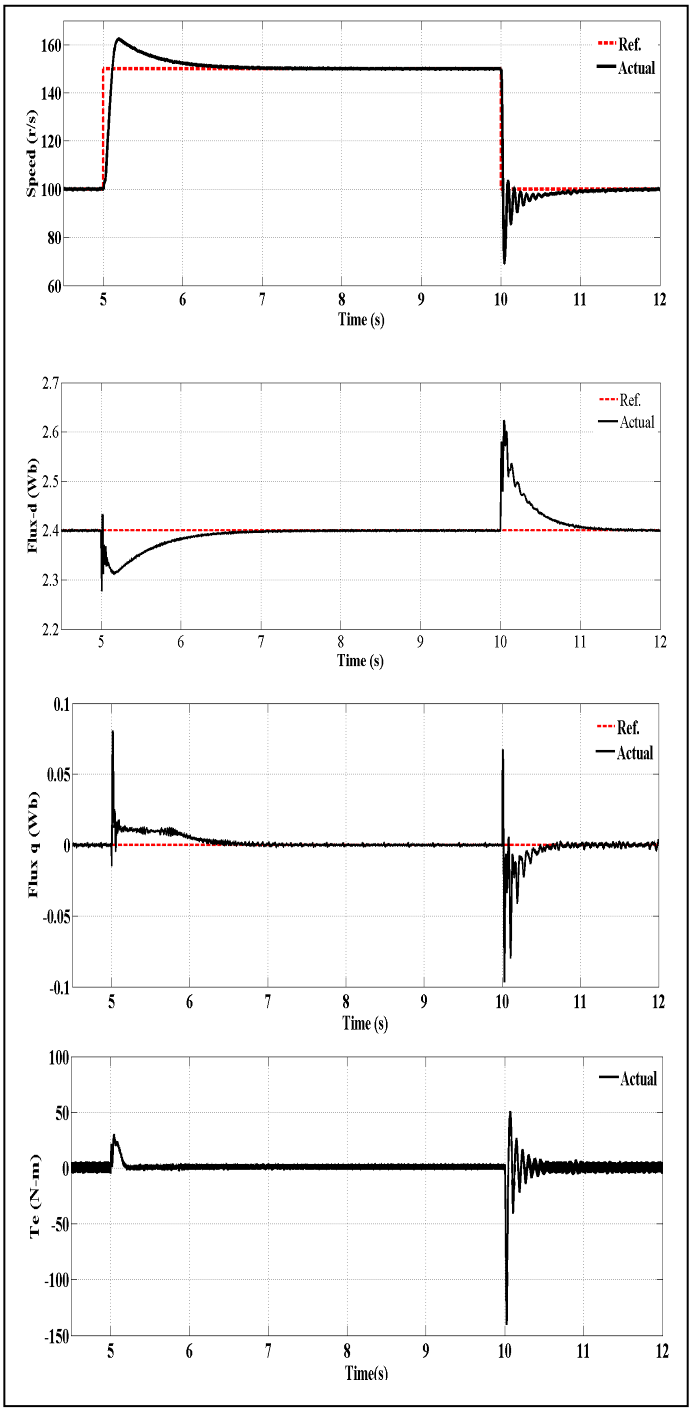

Fig 5: Enlarged Speed, flux and electromagnetic torque Vs Time with step change in speed reference.

The proposed controller is shown to achieve perfect speed reference tracking successfully. It is interesting to note that the flux ϕ_{ds} & ϕ_{qs} have also been recovered back to their reference value 2.4 Wb and 0 Wb, respectively.

V. CONCLUSION

In this paper, a full state feedback control of IM MIMO system is addressed based on optimal LQR theory. The proposed control law mimics the classical proportional-integral control law. Therefore, it has the goodness of the classical proportional integral control as well as the comfort of modern time domain robust control. The effectiveness of the proposed controller is verified in MAT LAB simulation environment. The results obtained are found satisfactory.

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