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Multivariable Offset-free Model Predictive Control for Quadruple Tanks System

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***Abstract* -- The design and implementation of a robust multivariable model predictive control (MPC) on a quadruple tanks system is addressed in this paper. Mismatch between the MPC's model and the process may cause constraint violation, non-optimized performance and even instability. It is the objective of this paper to offset-free control the process in the presence of constraints and model mismatch. It is shown in this paper how this model mismatch is compensated by augmented state disturbances, and also how the steady state error is eliminated. In the proposed method, an observer is designed to estimate the disturbances and states. The results show how the proposed control method increases the robustness of the model predictive controller in simulation and in real time implementations on a new quadruple tanks system proposed in this work together with techniques designed to identify the parameters of this novel plant.**

***Index Terms*-- Model Predictive Control, Offset-free Control, Robust Control, Parameter Uncertainty.**

I. INTRODUCTION

Closed-loop performance of systems controlled by model predictive controller depends on model's accuracy and disturbances. In practice, model mismatch and unmeasured disturbances can lead to a steady-state offset unless precautions are taken during the control design. There exist in the literature a number of optimization programming algorithms for offset free MPC, [1-3]. These algorithms, however, each considers different disturbances models and makes different assumptions in order to guarantee offset free control. Since there are different mismatch models or unmeasured disturbances, closed loop performance is sensitive to the type of disturbance model used for a given plant [4, 5]. Generally three basic methods are presented in order to eliminate the steady state offset. The first approach incorporates the integration of the tracking error with the process model. The drawback of this method is that the increase in the number of state variables due to the augmentation increases the computational cost of the optimization problem, especially in large scale systems. The requirement of an anti-windup system is the second drawback of this method. The second approach involves a velocity form of a state-space model to achieve offset-free control. The main disadvantage here is that the state's dimension is increased, which in turn increases the computational cost in the dynamic optimization. The third approach involves modifying the plant model to include a disturbance model. These disturbances can be estimated from the measured process output if the augmented system is observable. The augmented disturbance can be constant, ramp, periodic or stochastic [6], but in most cases, it is assumed to be constant. In order to eliminate the effects of the estimated disturbances, a target generator is used to modify the steady state target for the controller. As a result, zero steady-state offset output tracking is obtained by a linear MPC if the process is not strongly nonlinear over a wide range. Although this approach eliminates anti-windup requirements, its disadvantage is the demand for designing a disturbance model and an estimator. The demand for an observer makes the controller inapplicable to unstable processes because the observer poles contain the unstable poles of the process model. On the other hand, a system with a disturbance model may lead to unacceptable performances if a disturbance enters the process from somewhere else.

In this research, an enhanced scheme based on the third method is developed to design an offset-free model predictive controller. Although the third method generally uses both the state disturbances and output disturbances, it is shown in this paper that for a class of processes, employing only state disturbances satisfies the conditions that approaches offset-free control. As the result of eliminating the output disturbances in the proposed method, the computational time needed to solve multi parametric programming should be significantly reduced.

This paper also introduces a new type of nonlinear MIMO system; a modified quadruple tanks system (QTS). The plant is used to verify the proposed method and an offset free MPC control algorithm is designed for the system. The theory is then verified by simulation and real time implementation. For this purpose, a laboratory QTS is built and equipped with sensors, actuators, and data acquisition hardware. The system is interfaced and controlled by a PC equipped with LabVIEW®. MATLAB® is also installed and used to solve the multi-parametric program. In this paper, parameter identification techniques for the proposed QTS were also developed.

Classical QTS configurations consist of two upper tanks and two lower tanks. Many researchers simulated classical QTSs as a nonlinear multivariable benchmark to illustrate the benefits of their proposed controller algorithms. For instance, [7] proposed a sliding mode controller and applied it on a

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classical QTS to verify his design. In [8], a classical QTS is used for verification of a proposed decentralized robust control. In [9], the classical QTS is controlled by distributed model predictive control. In these studies, many nonlinear and time varying properties of the classical QTS were not considered in simulations. These system perturbations appear all the time during experimental testing, especially when variable tests are performed. A few researchers verified their proposed control algorithm on an experimental QTS [10-13].

The modified QTS configuration is introduced in this paper to increase the nonlinearity aspect of the process. The difference between the classical and modified configurations is that in the new configuration all the four tanks are adjacent to each other. This affects the dynamics of the plant such that the dynamic equations of the modified configuration include more intense nonlinearity. Figure 1 illustrates the two configurations; Fig. 1(a) shows the classical QTS configuration while Fig. 1(b) shows the modified configuration used in this paper.

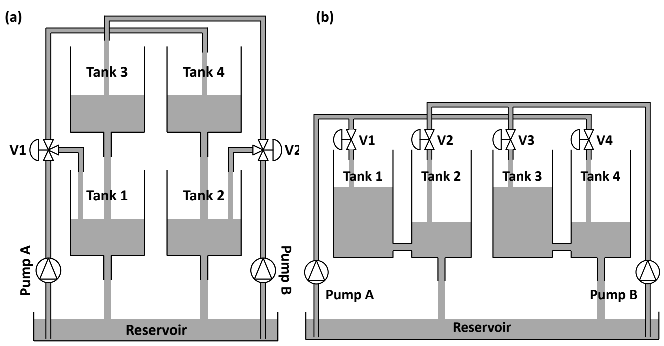

Fig. 1 Schematic of quadruple tanks system; (a) classical configuration, and (b) modified configuration.

II. QUADRUPLE TANKS SYSTEM AND EXPERIMENTAL SET-UP

The quadruple tank system is a multivariable process that has been used to show the results of different control strategies. The aim is to control the tank's water levels while simultaneously eliminating the water level offsets. As illustrated in Fig. 1(b), the water is pumped to tanks T_1 and T_4 by pump A, and to tanks T_2 and T_3 by pump B. A reservoir is located below the tanks to collect the outgoing water from tanks T_2 and T_4. There is also a hole in the wall between tanks T_1 and T_2, with a cross section of a_1, and another in the wall between the tanks T_3 and T_4, with a cross section of a_3. There are also two outlet holes, one in T_2 with a cross section of a_2 and one in T_4 with a cross section of a_4, that direct water to the reservoir. The valves V_2 and V_3 adjust the rate of water flowing into T_2 and T_3. Valves V_1 and V_4 are responsible for similar adjustments, which determine the rate of water flowing into T_1 and T_4. The valves are fixed, and are unchanged throughout the experiment.

Figure 2 shows the experimental quadruple tanks system that is constructed for this work in the Center for Research on Applied Electronics at the University of Malaya.

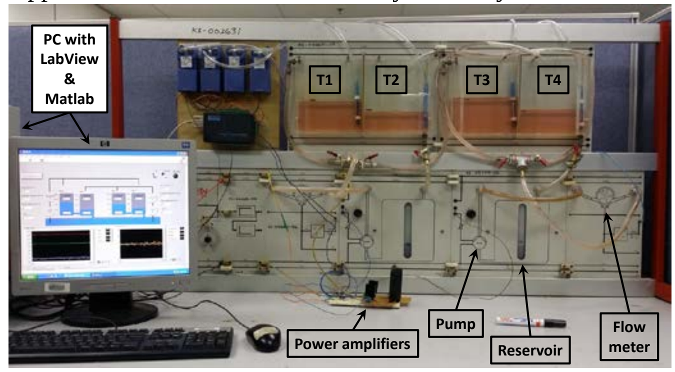

Fig. 2 Experimental set-up of a quadruple tank system.

The level of water in each tank is measured by a differential pressure measurement sensor. A vertical solid pipe is installed in each tank, with an open lower end, and its upper end is connected to its dedicated differential pressure sensor via a flexible pipe. When the water level in the tank increases, the volume of the air in the pipe will decrease, inevitably increasing the air pressure in the pipe. The differential pressure sensor measures the pressure difference between the pipe and the room's air pressure, and produces an output current of 4-20 mA.

The data acquisition system used is USB-4716 from ADVANTECH, a multifunction module consisting of analog and digital inputs and outputs. A 220Ω resistor converts the output current of the pressure sensor to a voltage signal measurable by an analog to digital converter. The multifunction module is connected to the USB port of a computer, and the measured data is transferred to the PC every four seconds. Figure 3 shows the water level sensor and the data acquisition hardware.

A. Nonlinear model

The nonlinear model of the QTS is obtained by Mass balances and Bernoulli's law as in (1), where the system's constraints, have been identified by analysing the physical dimensions of the system and its limits.

$$
\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2g(h_1-h_2)} + \frac{(1-\gamma_a)}{A_1}q_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_1}{A_2}\sqrt{2g(h_1-h_2)} + \frac{\gamma_b}{A_2}q_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2g(h_3-h_4)} + \frac{(1-\gamma_b)}{A_3}q_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{a_3}{A_4}\sqrt{2g(h_3-h_4)} + \frac{\gamma_a}{A_4}q_1
\end{aligned}
\qquad (1)
$$

where h_i is the water level and A_i is the cross-section of tank T_i, $i = 1 \cdots 4$. The flows of water, pumped by P_a and P_b, are denoted by q_1 and q_2 respectively.

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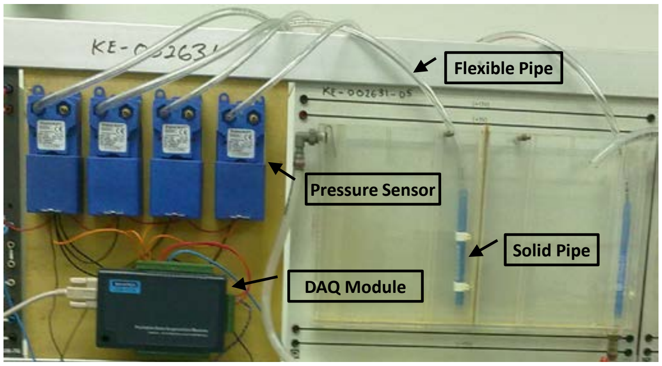

Fig. 3 Sensors and data acquisition hardware.

For tanks' heights (h_1, h_2, h_3, and h_4), the minimum constraint would be the minimum level of the water in the tank that cannot flow out of the tank and the maximum constraint would be the maximum level of the water after which water will overflow from the tank. As for the flow constraints (q_1 and q_2), the minimum was decided experimentally by identifying the minimum flow that does not cause any bubbles in the system. The maximum flow constraint is the maximum flow that the pump can produce.

The rate of water flow of tanks T_4 and T_1 is denoted by γ_a and the rate of water flow of tanks T_2 and T_3 is denoted by γ_b. The adjustments of the valves determine the values of γ_a and γ_b.

B. Parameter identification

The cross sections of the tanks are easily measured by measuring its physical dimensions, but the effective values of the cross sections of outlets of the tanks ($a_1, \cdots, a_4$) are not easily measured. Contrary to a_1 and a_3 that are fixed, a_2 and a_4 are adjustable via the valves placed at the outlets of tanks T_2 and T_4.

The most accurate measurement method for cross areas of the outlets is parameter identification using input-output data. The process is modeled by a gray box consisting of the nonlinear model presented in (1) with unknown parameters $a_1, \cdots, a_4$. Since it is a nonlinear multivariable model, the general method for $a_1, \cdots, a_4$ parameter estimation is too complex. In order to simplify the estimation method, we used a technique that converts the multivariable system with four unknown parameters to four systems, each having only one unknown parameter. This experimental technique is described in the remainder of this section.

The objective of the first experimental technique is to estimate a_2 and a_4. The quadruple tank system built during the course of this project is designed such that there is a removable common wall between tanks T_1 and T_2. When the common wall is removed, tanks T_1 and T_2 are converted into one tank, with a cross section $A_1 + A_2$. First, this tank is filled with water. While no more water is poured into the tank (as pumps are off), and the water is going out through the outlet to the reservoir, the data acquisition hardware starts recording and saving the signal of the water level sensor located in tank T_1. The dynamic equation of this system is described by (2):

$$\frac{dh}{dt} = -\frac{a_2}{A_1+A_2}\sqrt{2gh} \qquad (2)$$

where h is the water level in the merged tank, $A_1 + A_2$ is the cross section of the merged tank, g is the gravitational acceleration, while a_2 is the cross section of the outlet of tank T_2.

The underlying problem is to estimate a_2 using (2), and acquire the water level h. The least mean square method is used to obtain the optimum value of a_2 that most closely matches the model represented by (2) for the water level data. The optimum value for a_2 was determined to be $a_2 = 6 \times 10^{-6} m^2$.

A similar experiment on tanks T_3 and T_4 is carried out to estimate a_4, and a value of $a_4 = 6.7 \times 10^{-6} m^2$ is determined to be the optimum value that produces the best match for the model and the acquired data.

The second experimental technique is designed to estimate a_1 and a_3. Here, the process is simplified to a zero input single output process. Tank T_1 is filled by pump P_a, while pump P_b is off and the outlet of tank T_2 is shut. When the water level in tank T_1 is high enough, pump P_a is turned off and data logging starts until the water level in both T_1 and T_2 are the same. Figure 4 shows the simplified schematic of the process when both pumps are off. During the data logging, the dynamics of the process can be modeled as,

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2g(h_1 - h_2)} \qquad (3)$$

where h_1 and h_2 are the water levels in tanks T_1 and T_2, respectively.

The least mean square method is used to obtain the optimum value for a_1 that most closely matches the model (3) to the water level data. The optimum value for a_1 was determined to be $a_1 = 8.5 \times 10^{-6} m^2$. A similar method is used to estimate a_3. Table 1 shows the parameter values of the quadruple tanks system, which is constructed for this project.

TABLE I
PARAMETER VALUES OF THE QUADRUPLE TANK SYSTEM

Parameter	*Value*	*Unit*
A_1	*0.0033*	m^2
A_2	*0.0033*	m^2
A_3	*0.0033*	m^2
A_4	*0.0033*	m^2
a_1	8.5×10^{-6}	m^2
a_2	6×10^{-6}	m^2
a_3	8.5×10^{-6}	m^2
a_4	6.7×10^{-6}	m^2
g	*9.8*	m/s^2

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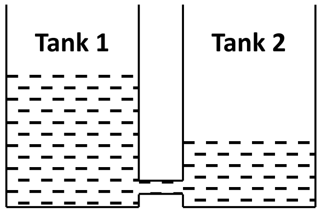

Fig. 4 Experiment to estimate a_1.

In addition to the parameters defined in Table 1, there are two uncertain parameters, γ_a , γ_b. Their nominal values are $\gamma_a = 0.25$ and $\gamma_b = 0.25$.

C. Steady state analysis

At steady state, since the levels of water in the tanks remain constant, we have:

$$\frac{dh_i}{dt} = 0 \quad i = 1 \cdots 4 \tag{4}$$

In this state, the inputs should be constant. Therefore, with the justifiable conclusion that inputs are constants, one can use (4) in (1) and obtain the steady state water levels,

$$\begin{aligned}
h_1^0 &= ((\frac{(1-\gamma_a)}{a_1} q_1^0)^2 + (\frac{1-\gamma_a}{a_2} q_1^0 + \frac{\gamma_b}{a_2} q_2^0)^2)/(2g) \\
h_2^0 &= (\frac{(1-\gamma_a)}{a_2} q_1^0 + \frac{\gamma_b}{a_2} q_2^0)^2/(2g) \\
h_3^0 &= ((\frac{(1-\gamma_b)}{a_3} q_2^0)^2 + (\frac{(1-\gamma_b)}{a_4} q_2^0 + \frac{\gamma_a}{a_4} q_1^0)^2)/(2g) \\
h_4^0 &= (\frac{(1-\gamma_b)}{a_4} q_2^0 + \frac{\gamma_a}{a_4} q_1^0)^2/(2g)
\end{aligned} \tag{5}$$

where the steady state water level in tank T_i is denoted by h_i^0 , $i = 1 \cdots 4$, and the final values of flow of water, pumped by P_a and P_b, are denoted by q_1^0 and q_2^0 respectively.

D. Linear model

The linearized model at a given steady state operating point is determined by using the approximate equation,

$$\sqrt{h_1 - h_2} \cong \sqrt{h_1^0 - h_2^0} + \frac{(h_1 - h_2 - (h_1^0 - h_2^0))}{(2\sqrt{h_1^0 - h_2^0})} \tag{6}$$

The state and input variables are defined as below:

$$\begin{aligned}
x_i &\triangleq h_i - h_i^0 \ , i = 1, \cdots, 4 \\
u_i &\triangleq q_i - q_i^0 \ , i = 1,2
\end{aligned} \tag{7}$$

The continuous linear state space matrices are defined as (9), where

$$\begin{aligned}
\tau_1 &= \frac{A_1}{a_1}\sqrt{\frac{2(h_1^0 - h_2^0)}{g}}, \tau_2 = \frac{A_2}{a_2}\sqrt{\frac{2h_2^0}{g}} \\
\tau_3 &= \frac{A_3}{a_3}\sqrt{\frac{2(h_3^0 - h_4^0)}{g}}, \tau_4 = \frac{A_4}{a_4}\sqrt{\frac{2h_4^0}{g}}
\end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{dx(t)}{dt} &= \begin{bmatrix} \frac{-1}{\tau_1} & \frac{1}{\tau_1} & 0 & 0 \\ \frac{A_1}{A_2\tau_1} & -\frac{1}{\tau_2} - \frac{A_1}{A_2\tau_1} & 0 & 0 \\ 0 & 0 & \frac{-1}{\tau_3} & \frac{1}{\tau_3} \\ 0 & 0 & \frac{A_3}{A_4\tau_3} & -\frac{1}{\tau_4} - \frac{A_3}{A_4\tau_3} \end{bmatrix} x(t) \\
&+ \begin{bmatrix} \frac{1-\gamma_a}{A_1} & 0 \\ 0 & \frac{\gamma_b}{A_2} \\ 0 & \frac{1-\gamma_b}{A_3} \\ \frac{\gamma_a}{A_4} & 0 \end{bmatrix} u(t) \\
y(t) &= x(t)
\end{aligned} \tag{9}$$

The discrete state space is obtained using the sampling period $T_s = 1\ sec$ and the parameter values given in Table 1 around the steady state operating point:

$$\begin{aligned}
q_1^0 &= 8.33 \times 10^{-6}\ m^3/s \\
q_2^0 &= 7.5 \times 10^{-6}\ m^3/s \\
h_1^0 &= 0.166\ m \\
h_2^0 &= 0.115\ m \\
h_3^0 &= 0.099\ m \\
h_4^0 &= 0.058\ m
\end{aligned} \tag{10}$$

The linear model of the QTS around the steady state operating point (10) is obtained as:

$$\begin{aligned}
x(k+1) &= \begin{bmatrix} 0.9815 & 0.0184 & 0 & 0 \\ 0.0184 & 0.9711 & 0 & 0 \\ 0 & 0 & 0.9795 & 0.0203 \\ 0 & 0 & 0.0203 & 0.9601 \end{bmatrix} x(k) \\
&+ \begin{bmatrix} 225.2 & 0.7031 \\ 2.109 & 74.65 \\ 0.7778 & 224.9 \\ 74.23 & 2.333 \end{bmatrix} u(k) \\
y(k) &= x(k)
\end{aligned} \tag{11}$$

The water level should be less than the height of the tank, and the tanks should not be empty. Since the water level sensors cannot sense a water level of less than 2 cm, the minimum water level threshold is set to be 2 cm. The maximum value of input is constrained by the maximum power of the water pumps, while the minimum value of input is set to obtain enough water pressure to prevent gas bubbles

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creation in the pipes. Therefore, the constraints on the quadruple tanks system is summarized as,

$$
\begin{aligned}
&0.02 < h_1 < 0.19\\
&0.02 < h_2 < 0.19\\
&0.02 < h_3 < 0.19\\
&0.02 < h_4 < 0.19\\
&4.15 \times 10^{-6} < q_1 < 13.85 \times 10^{-6}\\
&4.15 \times 10^{-6} < q_2 < 13.85 \times 10^{-6}
\end{aligned} \tag{12}
$$

III. Proposed Offset-free model predictive control

The main goal here is to control the level of the tanks T_2 and T_4. Offset-free techniques are employed to eliminate the steady state error. As a result, the robustness of the closed loop system against step-like disturbances and noisy measurement is tremendously improved.

In this paper, we only consider the state disturbance, and will show that the state disturbance model is capable of eliminating the offset created by the model's mismatch. Thus, the augmented model, including its state disturbance, is considered to be:

$$
\begin{aligned}
\begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} &= \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix}\begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k)\\
y(k) &= \begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} x(k) \\ d(k) \end{bmatrix}
\end{aligned} \tag{13}
$$

In order to compute the model predictive control signal, in each sampling period, first, the state and disturbance are estimated. A stable observer is necessary to estimate the unmeasured (in the case where disturbance is applied to the system) or virtual (in the case where virtual disturbance is used to compensate the model mismatch) disturbances. If it is not possible to measure all the states, the observer should make an appropriate estimate.

The main idea is to estimate the target values of system states and inputs. These will be used in a new system where the states are the difference between the states and the target states and the inputs are the difference between the inputs and the target inputs (δu). The constraints should be updated for this new system. Using model predictive techniques, optimum δu can then be obtained. Finally, the optimum input is calculated.

For this estimation to take place, the model needs to be observable. Observability conditions for the model of equation (13) are given in the following proposition:

Proposition 1. The augmented system (13) is observable if and only if (A, C) is observable, and B_d has a full column rank.

Proof: From the Hautus observability test, system (13) is observable if and only if $\begin{bmatrix} A^T - \lambda I & 0 & C^T \\ B_d^T & I - \lambda I & 0 \end{bmatrix}$ has a full row rank for all λ. According to a third set of columns, the first set of rows is linearly independent of the second. From the Hautus condition, the first set of rows is linearly independent for all λ if and only if (A, C) is observable. If $\lambda \neq 1$, the second set of rows is obviously linearly independent, while if $\lambda = 1$, the second set of rows is linearly independent if and only if B_d^T has full row rank. ■

Corollary 1. In order to satisfy the condition that $B_d \in \mathcal{R}^{n \times n_d}$ has a full column rank, the number of disturbances n_d should be less than or equal to the number of states n_x. i.e.$n_d \leq n_x$.

The observer shown in (14) is designed to estimate both states and disturbances from the measured outputs of y_m.

$$
\begin{aligned}
\begin{bmatrix} \hat{x}(k+1) \\ \hat{d}(k+1) \end{bmatrix} = &\begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix}\begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k)\\
&+ \begin{bmatrix} L_x \\ L_d \end{bmatrix}(-y_m(k) + C\hat{x}(k))
\end{aligned} \tag{14}
$$

Proposition 2. For any stable observer as given in (14), L_d has full row rank.

Proof: System (14) can be written as (15),

$$
\begin{aligned}
\begin{bmatrix} \hat{x}(k+1) \\ \hat{d}(k+1) \end{bmatrix} = &\begin{bmatrix} A + L_x C & B_d \\ L_d C & I \end{bmatrix}\begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k)\\
&- \begin{bmatrix} L_x \\ L_d \end{bmatrix} y_m(k)
\end{aligned} \tag{15}
$$

Since the observer is stable, it has no pole at (1 , 0). Consequently $\begin{bmatrix} A - I + L_x C & B_d \\ L_d C & 0 \end{bmatrix}$ is nonsingular. As a result, the second set of rows has to be a full row rank, and as a necessary condition, L_d has to also be full row rank. ■

Proposition 3. Assume that the system is modeled by (13), the observer (14) is stable, and the number of disturbances is chosen to be the same as the number of measured outputs. i.e. $n_d = p$. System (13) at steady state satisfies:

$$
\begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}\begin{bmatrix} \hat{x}_\infty \\ u_\infty \end{bmatrix} = \begin{bmatrix} -B_d \hat{d}_\infty \\ y_\infty \end{bmatrix} \tag{16}
$$

where u_∞, y_∞, $\hat{x}_\infty$ and $\hat{d}_\infty$ denote input, output, estimated state and estimated disturbance as $t \longrightarrow \infty$.

Proof: The stability of (14) and convergence of the estimated disturbance $\hat{d}(k)$ imply $L_d(-y_\infty + C\hat{x}_\infty) = 0$. Assuming $n_d = n_y$ implies that L_d is a square matrix, and by

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Proposition 2 it is nonsingular. Therefore, $-y_\infty + C\hat{x}_\infty = 0$, which is the second set of rows of (16). From (14), we have $(A - I)\hat{x}_\infty + B_d\hat{d}_\infty + Bu_\infty = 0$, which is the first set of rows of (16). ■

The stability of the observer is critical. The observer poles affect the closed loop response in transient time. After estimation, the target values of states and inputs i.e. $[\bar{x}(k) \quad \bar{u}(k)]^T$ are updated by solving the following equation:

$$\begin{bmatrix} A-I & B \\ HC & 0 \end{bmatrix} \begin{bmatrix} \bar{x}(k) \\ \bar{u}(k) \end{bmatrix} = \begin{bmatrix} -B_d\hat{d}(k) \\ r(k) \end{bmatrix} \quad (17)$$

where H is the matrix which defines the tracking outputs z by $z = Hy$.

Proposition 4. There exists a unique solution to (17) for $[\bar{x}(k) \quad \bar{u}(k)]^T$, provided that *(A,B)* is controllable and *HC* has full row rank.

Proof: The first set of rows of $\begin{bmatrix} A-I & B \\ HC & 0 \end{bmatrix}$ has full row rank because *(A, B)* is controllable, and the second set of rows has full row rank according to the assumption. In addition, the second set of columns ensures that the first and second set of rows is linearly independent of each other. ■

The variables used in the optimization problem are defined as follows.

$$\begin{aligned} \delta x_{k+i|k} &\triangleq x_{k+i|k} - \bar{x}(k) \\ \delta u_{k+i|k} &\triangleq u_{k+i|k} - \bar{u}(k) \end{aligned} \quad (18)$$

Finally, the following optimization problem is solved;

$$\min_{\delta u_0 \cdots \delta u_{N-1}} \left\| \delta x_{k+N|k} \right\|_P^2 + \sum_{i=0}^{N-1} (\left\| \delta x_{k+i|k} \right\|_Q^2 + \left\| \delta u_{k+i|k} \right\|_R^2) \quad (19)$$

subject to:

$$\begin{aligned} \delta x_{k+i+1|k} &= A\delta x_{k+i|k} + B\delta u_{k+i|k} \\ E\delta x_{k+i+1|k} + L\delta u_{k+i|k} &\leq M - E\bar{x}(k) - L\bar{u}(k) \end{aligned} \quad (20)$$

where $\|x\|_M^2 \triangleq x^T M x$, $Q \geq 0$, $R > 0$, and P is the solution of the Riccati equation. The variable $\delta x_{k+i|k}$ denotes the predicted variable at time $i + k$ obtained by starting from $\delta x(k)$. Matrices E, L and M are obtained by (7), (10) and (12), and indicate the constraints of the plant and controller.

Therefore, the output of the system, which is controlled by the model predictive controller, reaches the target $r(k)$ as $k \longrightarrow \infty$ under the following conditions [14]:

- The closed loop system is asymptotically stable.
- The process is controllable and observable.
- The number of disturbances is equal to the number of outputs.
- Augmented system is observable.
- Constraints are not active at steady state.

Finally, to implement the proposed offset-free model predictive control, the following algorithm summarizes the overall control implementation:

Algorithm:

1) Initialize $\hat{x}$, $\hat{d}$, $\bar{x}$ and $\bar{u}$
2) Measure outputs $Y(k)$
3) Subtract the operating point $y(k) = Y(k) - h^0$
4) Estimate $\hat{x}(k)$ and $\hat{d}(k)$ by (16)
5) Compute $\bar{x}(k)$ and $\bar{u}(k)$ by (17)
6) Compute $\delta x(k)$ by (18)
7) Update constraints using (20)
8) Solve multi-parametric programming (19) and obtain δu^*
9) Compute controller output $u(k) = \delta u_0^* + \bar{u}(k)$
10) Add the operating point and apply to the process $U(k) = u(k) + u^0$
11) If not end of experiment, go to step 2.

IV. Simulation Results

The closed loop system is simulated, taking into consideration uncertainties in γ_a and γ_b as shown in Table 2. Figure 5 shows the states and control signal, while the inputs are subject to a constraint of $-2 < u_i < 2 \quad i = 1,2$. Figure 5(a)-(b) shows that the controller is able to move the tracking states (water levels of tanks T_2 and T_4) from its initial condition to the origin, and when the references change (at *t* = 500 sec), these states will track the references. In cases where model mismatch is absent, these states converge to the origin, but the model mismatch causes a non-zero steady state. Since there are two inputs in this case study, the steady state error of only two outputs are eliminated. Figure 6 shows the control signals are indeed in range. Note that there is a reference jump at *t* = 500 sec, which causes a transient error. By the next experiment, it is shown that the offset-free technique employed in this paper is able to eliminate this steady state error.

For comparison purposes, the system is simulated by LQR and the results are shown in Figs. 7 and 8. As shown in Fig. 7(a)-(b), there are steady state errors in the tracking outputs, and it is not offset free. Fig 8(b) shows that the control signal violated the upper constraint.

TABLE II
MISMATCHED PARAMETERS

Parameter	*Plant*	*Model*
γ_a	*0.4*	*0.25*
γ_b	*0.35*	*0.25*

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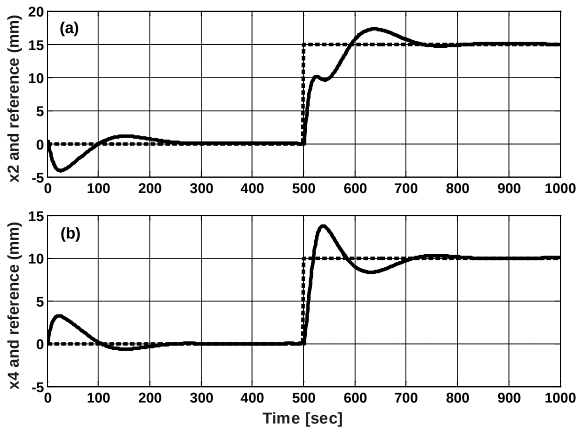

Fig. 5 Proposed MPC tracking outputs (solid) and corresponding references (dashed), (a) Tank 2 (b) Tank 4

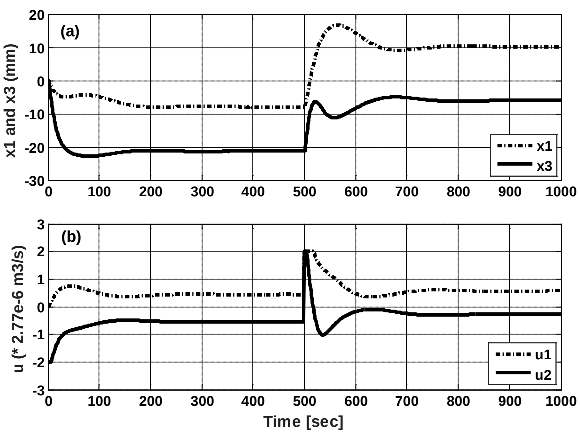

Fig. 6 Proposed MPC results; (a) state variables x_1 and x_3 (b) Control signals

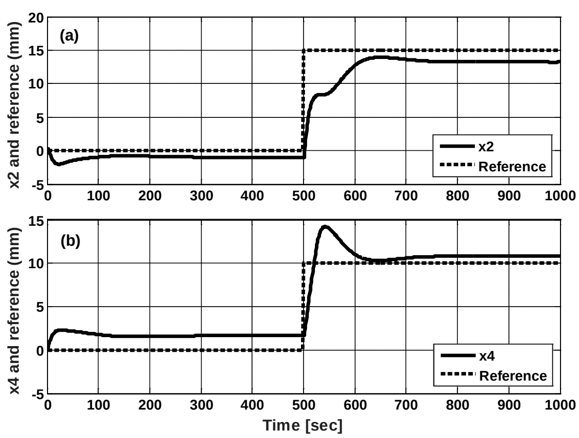

Fig. 7 LQR tracking outputs (solid) and corresponding references (dashed), (a) Tank 2 (b) Tank 4

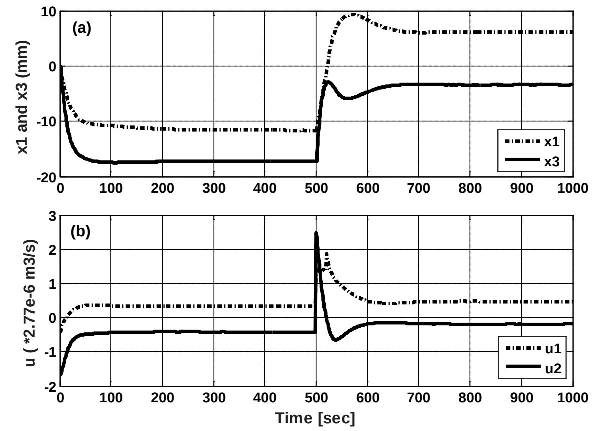

Fig. 8 LQR results; (a) state variables x_1 and x_3 (b) Control signals

V. EXPERIMENTAL RESULTS

The offset-free model predictive controller described in section III is applied to the developed quadruple tanks system. The objective is that the water levels of tanks T_2 and T_4 track the references. Model mismatch affects not only the linear system but also the operating point. Due to the model mismatch, $h_i^0, i = 1, \cdots, 4$ are not the steady state water level when the inputs are permanently equal to q_i^0 , $i = 1, 2$. Therefore, in practice, the origin is not the equilibrium point for a linearized system. The combination of LabVIEW and MATLAB creates a powerful software tool that allows the implementation of the controller on a PC. In order to obtain a powerful human machine interface, the controller is programmed using the LabVIEW programming language. This work uses the Multi Parametric Toolbox for MATLAB [15],[16]. The controller runs under LabVIEW but calls a MATLAB function from within and uses the Multi Parametric Toolbox to solve the optimization problem in each sampling time.

First, a reference signal of 30mm were given to both tanks T_2 and T_4 operating under the proposed MPC scheme. The measured water levels of tanks T_2 and T_4 are depicted in Fig. 9, which shows that the proposed controller succeeded in reference tracking and that the water level in both tanks followed the given reference with a slight steady state error in tank T4. The MPC output signal to the pumps, the resulting water flow and measured water level in tanks T1 and T3 are depicted in Figs. 10, 11 and 12 respectively.

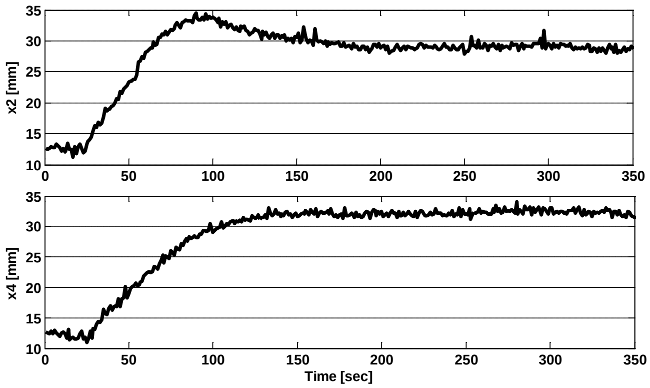

Fig. 9 MPC Measured water level in Tank T_2 and Tank T_4 respectively.

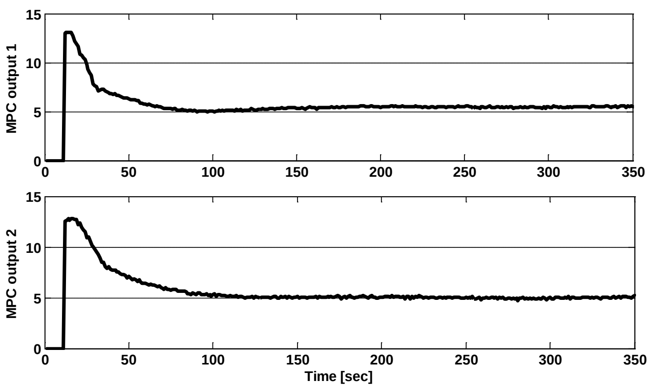

Fig. 10 MPC controller outputs to pump Pa and Pb respectively.

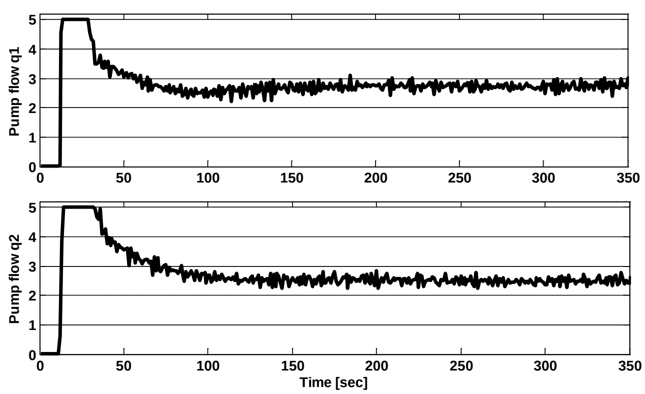

Fig. 11 MPC resulting flow of pump Pa and Pb respectively.

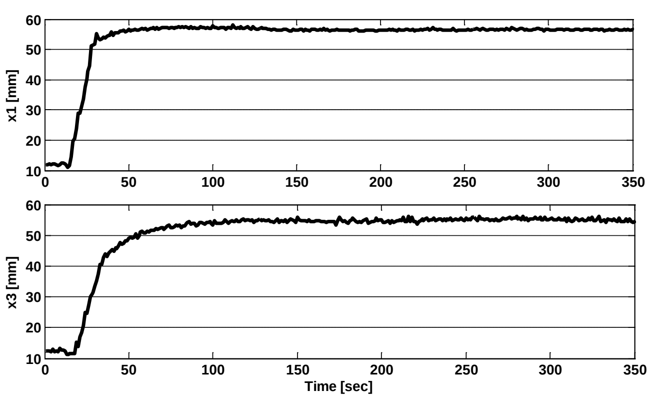

Fig. 12 MPC Measured water level in Tank T_1 and Tank T_3 respectively.

To verify the performance of the designed controller, a conventional LQR controller is designed and implemented on the same system using the same reference signals. The measured water levels of tanks T_2 and T_4 are depicted in Fig. 13, which shows that the LQR controller successfully reaching the reference for tank T2 but in a slower fashion and with slightly more overshoot. As for tank T4, the LQR controller suffers from offset at steady state and it is also slower to reach steady state. The LQR output signal to the pumps, the resulting water flow and measured water level in tanks T1 and T3 are depicted in Figs. 14, 15 and 16 respectively.

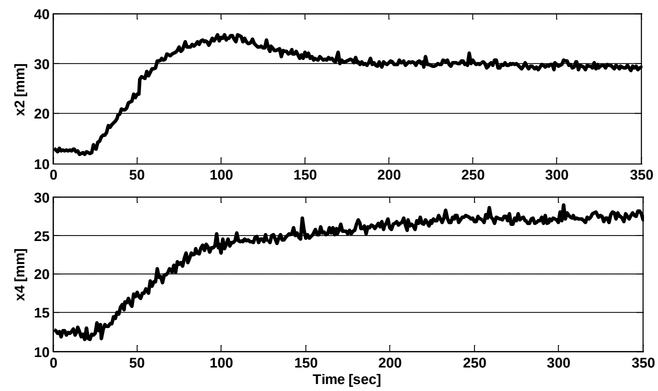

Fig. 13 LQR Measured water level in Tank T_2 and Tank T_4 respectively.

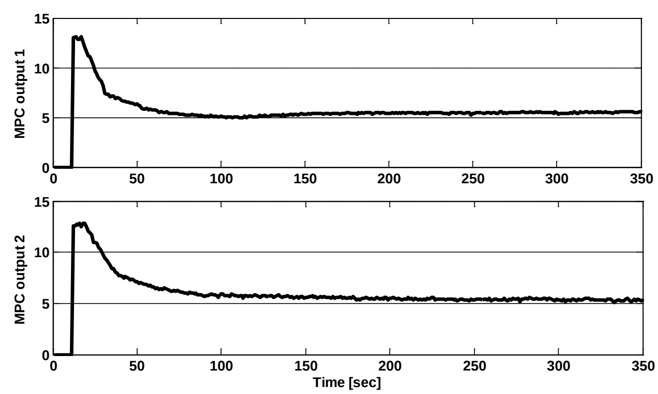

Fig. 14 LQR controller outputs to pump Pa and Pb respectively.

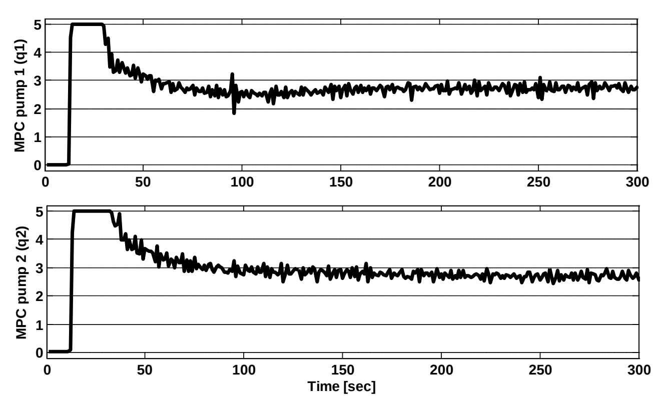

Fig. 15 LQR resulting flow of pump Pa and Pb respectively.

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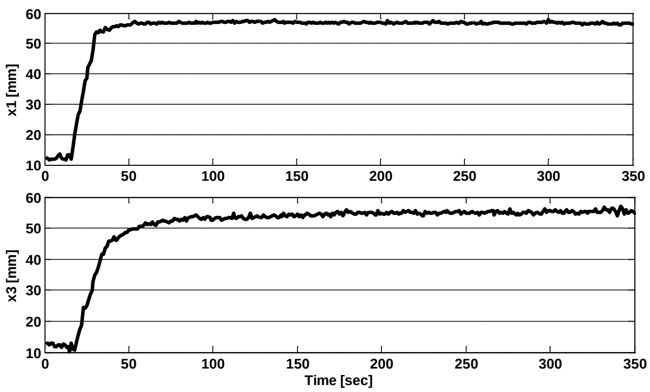

Fig. 16 MPC Measured water level in Tank T_1 and Tank T_3 respectively.

VI. Conclusion

The work of this paper develops a less computational method to achieve an offset free MPC control system. In addition, it addresses the practical implementation of the developed offset-free model predictive control on a non-classical and nonlinear multivariable quadruple tanks system. While the classical quadruple tanks system consists of two tanks up and two tanks down, proposed QTS consists of four tanks which are located besides each other. The connection between neighboring tanks makes the outlet water flow dependent on the difference of water level. Therefore, the describing differential equations of the proposed QTS configuration is more complex than classical ones.

Explicit MPC solves the optimization problem with constant constraints and saves the results in lookup tables. Therefore, in real-time execution it is not possible to change the constraints values, and constraint violation may happen. Whereas, the proposed offset-free MPC is able to solve the multi-parametric program with new constraints values at each control loop cycle. Thus, it is able to prevent constraint violations.

It is shown that the mismatch between the models and the plant is compensated by the augmented state disturbance. The proposed algorithm clearly presents the necessary steps to achieve the output of the controller. Both simulation and practical results show that the proposed offset-free MPC is successful to eliminate the steady state error even in noisy environment. The proposed controller was compared with a conventional LQR controller and the results show promise for the proposed MPC offset free controller.

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